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A Review of Fatigue Failure and Life Estimation Models: From Classical Methods to Innovative Approaches

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Abstract

This review paper encompasses a comprehensive exploration of fatigue failure and fatigue life estimation techniques which spans from the classical methods to new and innovative approaches. The paper looks into the limitations and advancements of these techniques and highlights their respective strengths and areas for improvement. Some of the models such as artificial neural networks and genetic algorithms exhibit clear advantages in terms of processing speed, accuracy, and adaptability to diverse materials and loading scenarios. For instance, in estimating fatigue life under multiaxial loading, the stress scale factor model emerges as a viable alternative to the critical plane-based approach, as this technique offers superior efficiency under both constant and variable amplitude loadings. Additionally, optimization algorithms such as artificial neural networks and genetic algorithms show promising potential in efficiently estimating fatigue life due to their rapid computational capabilities. Despite the notable successes achieved by these techniques, none of them can be ascribed as a universal model capable of accurately estimating the fatigue life of all materials across diverse operating conditions as each of the techniques possesses its unique strengths and weaknesses, thus, necessitating the study for a better understanding of their applicability. Hence, this paper serves as a valuable compilation of various fatigue analysis techniques, targeted at paving the way towards the development of a universal model capable of handling different materials and loading conditions.

Keywords: *Fatigue, critical plane, multiaxial, genetic algorithm, optimization algorithm, ant-stigmergy algorithms*

1. Introduction

Fatigue failure is a critical issue in the field of mechanical engineering, as it can lead to catastrophic consequences in structures and components subjected to cyclic loading [1]. It has been reported that approximately half of all mechanical breakdowns can be linked to fatigue [2]. Thus, understanding the underlying physics of fatigue is crucial for establishing cause-and-effect relationships capable of preventing such failure [3]. Experts in mechanical design and fatigue analysis have invested significant resources in crafting effective approaches to evaluate the safety of mechanical parts under dynamic or cyclic loads. They have done so through a combination of experimental research and predictive modelling techniques. [4-9]. In recent years, researchers have proposed innovative approaches to improve the accuracy and reliability of fatigue life estimation [10, 11]. These approaches include the development of multi-axial fatigue criteria, strain-based fatigue models, and probabilistic methods that account for uncertainties in material properties and loading conditions [11]. Additionally, advancements in computational modelling and simulation techniques have allowed for the analysis of fatigue behaviour at the microstructural level, providing insights into the mechanisms of fatigue failure [12].

Accurately predicting fatigue life in real-world situations presents a considerable challenge due to the numerous factors at play. Therefore, a thorough

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examination of mechanical parts under cyclic loading is essential to prevent unforeseen and catastrophic failures [13]. The effectiveness of fatigue estimation methods relies on their capability to consider various elements such as non-zero superimposed static stresses, multiaxial stress environments, and stress concentration impacts [14].

Estimating fatigue life under cyclic and random multiaxial loading is particularly intricate because damage accumulation relies on variations across all stress components throughout the loading period [14, 15]. Thus, fatigue assessment methods need to be adjusted to integrate experimental data obtained from tests conducted according to relevant standard codes, as this will enhance the accuracy of fatigue predictions [5, 13, 14, 16, 17]. Standard codes, such as those provided by organizations like ASTM International and ISO, define specific testing procedures and protocols for evaluating the fatigue behaviour of materials and components. These codes outline the test configurations, loading conditions, and data collection methods to ensure consistency and comparability of results. By conducting tests according to these standard codes, researchers and engineers can obtain experimental data that represents realistic operating conditions and material responses. This data is essential for calibrating and validating fatigue assessment methods, improving the accuracy of fatigue predictions, and ensuring the reliability of mechanical designs. Examples of such standard codes include ASTM E647 for fatigue crack growth testing, ASTM E466 for tension-tension fatigue testing, and ISO 12106 for axial fatigue testing of metallic materials. Furthermore, through the simple utilization of linear elastic finite element (FE) models, fatigue damage can be accurately estimated from stress analysis results [18-20].

This paper presents various fatigue life estimation techniques, with a focus on newly proposed models that aim to address limitations in older approaches, particularly in the accurate estimation of fatigue life under specific loading or material conditions. However, it is recognized that no universally applicable model exists for predicting fatigue life across diverse loading and material scenarios [5, 21]. The development of a universally applicable model for predicting fatigue life is challenging due to several factors. First, fatigue life estimation depends on various elements such as non-zero superimposed static stresses, multiaxial stress environments, and stress concentration impacts. These factors introduce complexity and variability into the loading conditions, thus, making it difficult to capture all possible scenarios with a single model. Additionally, different materials exhibit unique fatigue behaviour, and their response to cyclic loading can vary significantly. Therefore, it is challenging to develop a single model that accurately captures the fatigue life of all materials. It is worth noting that the limitations arise from the need to consider material-specific characteristics, loading conditions, and the availability of experimental data for model calibration. Thus, the development of a universally applicable model requires further research and a better understanding of the complexities involved. This review paper also serves as a compilation of innovative concepts about fatigue failure and fatigue life estimation methods and its application could pave the way for the development of a widely accepted model capable of addressing a broad spectrum of fatigue estimation challenges.

2. Fatigue Failure

In engineering, fatigue is a term used to describe the failure of materials or engineering components when subjected or exposed to "cyclic or dynamic" loading that is below the yield strength of such material or component [22]. The term "fatigue" originates from the Latin word 'fatigare', signifying 'to tire'. In other words, the term describes the gradual weakening of the material over repeated loading cycles, which can arise from mechanical, thermal, or other forms of cyclic loading. Fatigue failure can manifest in two forms: LCF (low cycle fatigue) or HCF (high cycle fatigue). HCF typically occurs due to small elastic strains over a high number of cycles, and the stress causing the failure is a combination of mean stress and alternating stresses that are induced by mechanical or thermal loading at varying frequencies [23, 24]. Conversely, LCF is distinguished by significant plastic strains of low frequency, resulting in failure after a comparatively small number of cycles [25].

Fatigue cracking is a significant damage mechanism in structural components, and it results from fluctuating stresses that are below the material's ultimate tensile or yield strength [1, 26]. Unlike other failure modes where the original design strengths are exceeded, fatigue damage can occur at low stress without visible warning signs, thus, making this failure hazardous. In the case of a component exposed to cyclic loading, the fatigue life is determined by the number of stress or loading cycles necessary to initiate and propagate a crack to a critical size. This process typically occurs in three phases: initial slow crack initiation, followed by stable crack expansion, culminating in rapid fracture. The occurrence of fatigue damage at low-stress levels without visible warning signs poses a significant safety concern for mechanical parts under cyclic loading. Unlike other failure modes where the original design strengths are exceeded, fatigue failure can initiate and propagate even at stress levels below the yield strength of the material. This means that parts subjected to cyclic loading may experience fatigue damage without any visible indications, such as deformation or cracking until a critical point is reached. This makes it challenging to detect and predict the remaining fatigue life of components, thus, increasing the risk of unexpected and catastrophic failures. Therefore, understanding fatigue behaviour and accurately estimating fatigue life are crucial for ensuring the safety and reliability of mechanical parts under cyclic loading.

During fatigue failure, dislocations accumulate near stress concentrations, thus, forming structures known as persistent slip bands (PSBs) after numerous loading cycles, and these PSBs formed, act as stress amplifiers for the initiation of micro-cracks. The initiated micro-cracks by the PSBs nucleate along high-shear stress planes, often at a 45-degree angle to the loading direction. Over time, the micro-cracks propagate perpendicular to the maximum tensile stress, where the few larger cracks dominate over the remaining smaller cracks. Continuous cyclic loading of such components leads to the continuous growth of the cracks and this will continue until the remaining intact section of the component can no longer withstand the load [1]. This critical point marks the point where fracture toughness is surpassed and sudden or rapid failure of the remaining material cross-section is inevitable. This stage is known as the third stage of fatigue failure, and it signifies the stage when the component experiences rapid overload fracture.

In fatigue analysis, uniaxial loading involves the application of cyclic stress in a single direction, and this provides the fundamental data on material response to cyclic loading [27, 28]. Fatigue testing under uniaxial conditions is straightforward in both its execution and analysis, thus, making it widely used [27, 29]. However, it oversimplifies real-world conditions, hence, limiting its representation and potentially leading to conservative or inaccurate fatigue life predictions for complex components [27]. Multiaxial loading on the other hand considers the interaction of stresses from multiple directions simultaneously, thus, accounting for complex stress states experienced by real-world structures and components [27, 30]. Conducting multiaxial fatigue analysis on a component helps to provide a comprehensive understanding of the material behaviour of such components under realistic loading conditions and allows for a more accurate fatigue life assessment of the component [30, 31]. However, it is more complex and requires sophisticated numerical methods and specialized testing equipment for experimental validation [27].

In real-world scenarios, the multiaxial nature of stresses introduces variability and complexity to loading conditions. For components under cyclic loading, multiaxial loads can manifest as either in-phase also known as proportional loads or out-of-phase also known as non-proportional loads, as frequently in a range of components and structures across industries such as aerospace, automotive, and power generation [32]. Even when subjected to uniaxial loads, multiaxial stresses may arise, even if the loading modes resemble in-phase loading due to geometrical constraints at notches. In cyclic-loaded components, non-proportional multiaxial fatigue damage may occur due to variations in principal axis directions induced by out-of-phase bending and torsional moments during loading [33]. However, strategies for handling very complex multiaxial variable loadings, particularly non-proportional loads, are not yet well established [34]. An established approach to estimating fatigue life under multiaxial stress entails simplifying the three-dimensional stress condition to an equivalent uniaxial state using suitable fatigue failure criteria [35-39], as illustrated in Figure 1.

The utilization of multiaxial fatigue failure criteria enables the translation of stress histories into equivalent stress, thus, enabling analysis similar to the uniaxial case. This approach facilitates the counting of fatigue cycles and the determination of how fatigue damage is accumulated of fatigue [40]. Figure 1 illustrates an algorithm that selectively identifies and reduces the three-dimensional (3D) stress state at any given time, and maintains the equivalent stress frequency range relative to the component's stress state. To meet these criteria, Macha [36, 37, 41], Łagoda and Ogonowski [37, 38] proposed a linear criterion grounded in the critical plane concept.

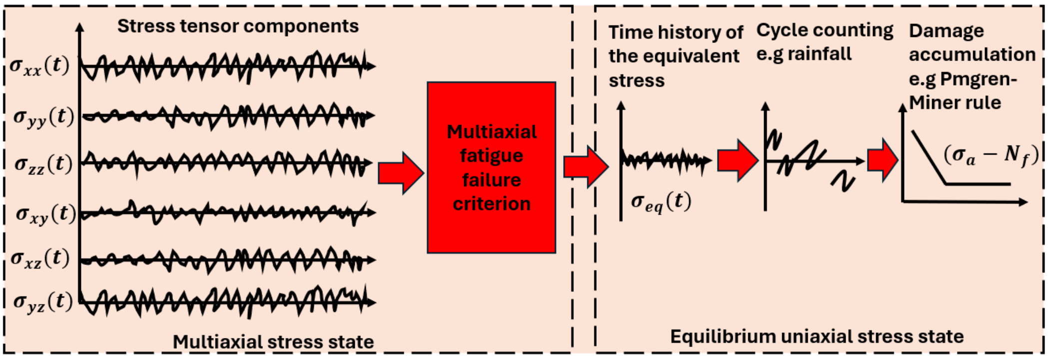

Figure 1. Transformation from multiaxial stress state to uniaxial stress state during the estimation of fatigue life [40].

This method effectively transforms the stress state from 3D to equivalent uniaxial stress, requiring separate cycle counts for damage calculation based on these equivalent stress histories [40].

3. Conventional Methods for Fatigue Life Estimation

In this section, conventional fatigue life estimation models employed over time are discussed. These models include equivalent stress, strain, and energy-based approaches. The symbols used for the different models and their descriptions are illustrated in Table A1 in the appendix.

3.1. Fatigue analysis models using equivalent stress

Equivalent stress-based models are widely utilized in fatigue analysis to simplify the complex stress states experienced by materials under cyclic loading conditions [42]. The concept of equivalent stress allows engineers to represent the combined effect of multiaxial stresses as a single, uniaxial stress component that captures the overall impact of various stress components on fatigue behaviour. By reducing the multidimensional stress state to a single value, equivalent stress-based models provide a practical and efficient approach to predicting the fatigue life of components and structures, nevertheless, their accuracy when dealing with non-proportional loading histories has raised some questions [43].

According to the proposed maximum normal stress criterion stress [37], fatigue in materials is primarily attributed to the range of maximum normal stress. Therefore, the hypothesis assumes that applying static effort to cyclic loading results in an equivalent stress range, as depicted in Equation (1). In this model, it is assumed that the critical plane contains the range of maximum normal stress, and the multiaxial stress state is simplified to uniaxial using the equivalent stress range $\Delta\sigma_{eq}$, which corresponds to the maximum normal stress range $\Delta\sigma_1$.

$$\Delta\sigma_{eq} = \Delta\sigma_1 \tag{1}$$

According to Tresca's hypothesis [37], fatigue in a material is attributed to the maximum shear stress range value. This criterion, as outlined in Equation (2), identifies two critical planes perpendicular to each other, as the maximum shear stress consistently arises in two perpendicular planes.

$$\Delta\tau_{13} = \frac{\Delta\sigma_{eq}}{2} = \frac{\Delta\sigma_1 - \Delta\sigma_3}{2} \tag{2}$$

The fatigue damage criterion proposed by Sine [44] and Sine [45] involves an equivalent stress derived from the alternating octahedral shear stress. Their investigation concluded that the octahedral shear stress criterion stated in Equation (3) lacks effectiveness in dealing with non-proportional loading.

$$\Delta\tau_{oct} + \alpha(3\sigma_h) = \beta \tag{3}$$

But,

$$\alpha = \frac{2\sqrt{2}}{3}\frac{\left(\sigma_{-1} - \frac{\sigma_0}{2}\right)}{\sigma_o} \tag{4}$$

$$\beta = \frac{\sqrt{3}}{2}\sigma_{-1} \quad (5)$$

The amplitude of the octahedral shear stress can be expressed using the alternating stress tensor components $\sigma_{ij,a}(i,j = 123)$ [46] as shown in Equation (6).

$$\Delta\tau_{oct} = \sqrt{\frac{1}{6}\left[\begin{array}{c}(\sigma_{11,a} - \sigma_{22,a})^2 + (\sigma_{22,a} - \sigma_{33,a})^2 + \\ (\sigma_{33,a} - \sigma_{11,a})^2 + 6(\sigma_{12,a}^2 + \sigma_{23,a}^2 + \sigma_{31,a}^2)\end{array}\right]} \quad (6)$$

A parameter similar to that of Sines's Equation (3) was proposed by Crossland [47] but with a modification that uses maximum hydrostatic stress $\sigma_{h\,max}$ in place of mean stress as depicted in Equation (7). This modification helps to address issues encountered when dealing with out-of-phase multiaxial loading [21].

$$\Delta\tau_{oct} + \alpha(3\sigma_{h\,max}) = \beta \quad (7)$$

The fatigue life parameter expressed in Equation (8) was proposed by Findley [48] and the expression was derived from the combination of shear stress and nominal stress on the plate with the highest parameter value.

$$[\tau_{a,n} + k\sigma_n]_{max} = f \quad (8)$$

But $\tau_{a,n} = \frac{\Delta\tau}{2}$ therefore, the parameter can be expressed as shown in Equation (9)

$$\left[\frac{\Delta\tau}{2} + k\sigma_n\right]_{max} = f \quad (9)$$

The material constants denoted as k and f, can be obtained through two fatigue loading tests: one involving alternating normal stress and the other involving pulsating normal stress.

A similar model to that of Findley's was proposed by McDiarmid [49, 50] as expressed in Equation (10). In the proposed model, the critical plane is identified as the one with the maximum shear stress range, although it yields results that are widely scattered.

McDiarmid [37, 38] introduced a model akin to Findley's, as illustrated in Equation (10). In this model, the critical plane is determined as the one with the maximum shear stress range, despite producing results that are considerably scattered.

$$\frac{\Delta\tau_{max}}{2t_{A,B}} + \frac{\sigma_{n,max}}{2\sigma_{uts}} = 1 \quad (10)$$

The Van Dang model also referred to as the endurance limit criterion, was introduced by Van [51]. He provided an expression, as shown in Equation (11), based on the concept of micro-stress developed within a critical volume of material. This criterion operates on two-scale approaches, permitting fatigue crack initiation at the grain level persistent slip bands (PSBs) due to alternating shear stresses. Dang Van fatigue failure criterion which focuses on micro-stress in the meridian plane, is depicted in Figure 2. In the Figure, the path of the safe stress is represented with a solid green line and the path of the stress leads to fatigue fracture is represented with a dashed blue/red line.

$$\tau(t) + a\sigma_h(t) = b \quad (11)$$

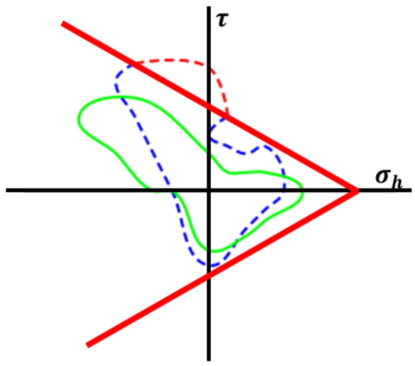

Figure 2. Fatigue failure criterion by Dang Van with a focus on micro-stress in the meridian plan [51].

3.2. Fatigue analysis models using equivalent strain

Fatigue analysis models that utilize equivalent strain are valuable tools in estimating the fatigue life of materials under cyclic loading conditions [52]. The models aim to simplify the complex strain states experienced by materials during fatigue loading by representing the overall impact of various strain components as a single, equivalent strain value [53]. Similar to equivalent stress models, equivalent strain models streamline the analysis process by reducing the multidimensional strain data to a single, uniaxial representation, providing a more straightforward approach to assessing fatigue damage and estimating the remaining useful life of components [53]. By transforming the multiaxial strain state into a scalar quantity the equivalent strain model enables a comprehensive assessment of strain accumulation and deformation, thus, aiding in the understanding of how different strain components contribute to fatigue failure [53].

Kandil and Brown [54] along with Brown and Miller [55, 56], introduced a parameter that is grounded on the assumption that fatigue failure is governed by the maximum shear strain plane and normal strain range. They proposed that fatigue failure occurs on the plane undergoing the highest shear strain range, as depicted in Equation (12).

$$S\Delta\varepsilon_n + \frac{\Delta\gamma_{max}}{2} = A\frac{\sigma_f - 2\sigma_{n,mean}}{E}(2N_f)^b + B\varepsilon_f(2N_f)^c \qquad (12)$$

The initial model introduced by Brown and Miller [55] underwent modification by Wang and Brown [57]. In their modification, as illustrated in Equation (13), they integrated extra parameters into the equation to accommodate the strain path effect.

The initial model introduced by Brown and Miller [41] underwent modification by Wang and Brown [43]. In their modification, as illustrated in Equation (13), they incorporated extra parameters into the equation to accommodate the strain path effect.

$$\frac{\Delta\acute{\gamma}}{2} = S\varepsilon_n^* + \frac{\Delta\gamma_{max}}{2} = (1 + v_e + (1 - v_e)S).\sigma_f'(2N_f)^b + (1 + v_p + (1 + v_p).S\varepsilon_f'.(2N_f)^c \qquad (13)$$

Consequently, the maximum Wang-Brown damage expression they obtained based using the maximum shear strain plane for the critical plane is as shown in Equation (14).

$$MDP_{WB} = \underset{t}{Max}(S\Delta\varepsilon_n^* + \gamma_a) \qquad (14)$$

3.3. Fatigue analysis models utilizing strain energy-based approach

Fatigue analysis models that utilize strain energy-based approaches are vital for predicting the fatigue life of materials under cyclic loading. These models leverage the concept of strain energy to evaluate the impact of cyclic loading on material deformation and damage accumulation, thus, providing a comprehensive framework for fatigue behaviour analysis. By quantifying the energy absorbed and dissipated during loading cycles, the technique allows engineers to understand how materials respond to fatigue loading and predict potential structural failures. These models are valuable in assessing fatigue crack growth rates and identifying critical areas prone to fatigue damage, allowing for targeted mitigation strategies and design improvements.

Introduced by Smith and Watson [58], the Smith-Watson Topper (SWT) model is a damage model that offers a comprehensive approach by integrating both the maximum nominal stress and the cyclic normal strain, as outlined in Equation (15) for fatigue analysis. This model is specifically valuable for analyzing multiaxial stress in materials susceptible to normal cracking. Within the SWT model, the critical plane is identified as the one with the highest normal stress, thus, facilitating the incorporation of mean stresses during multiaxial loading [58]. As a result of this, the model remains widely utilized for mean stress correction purposes.

$$\sigma_{n,max}\frac{\Delta\varepsilon_1}{2} = \frac{(\sigma_f')^2}{E}(2N_f)^{2b} + \sigma_f'\varepsilon_f'(2N_f)^{b+c} \qquad (15)$$

The Smith-Watson Topper damage parameter is expressed in Equation (16).

$$MDP_{SWT} = \underset{t}{Max}\{\varepsilon_{1,}\sigma_{n,max}\} \qquad (16)$$

When applying this damage parameter to GH4169 steel under uniaxial loading, the obtained outcome tends to be lower than the calculated result [59-62]. Consequently, while this method tends to overestimate multiaxial fatigue life, it provides a satisfactory life prediction for uniaxial [59].

In the modification to the Brown and Miller model, Fatemi and Socie [63] replaced the nominal strain with normal stress, as demonstrated in Equation (17). They also transformed the shear fatigue properties into uniaxial fatigue properties [63-65]. The normal stress term was modified to incorporate cyclic hardening that occurred during out-of-phase loading. Moreover, they incorporated cyclic hardening developed during out-of-phase loading into the normal stress term. This modification effectively addressed the mean stress by incorporating both the normal mean stress and the alternating normal stress along the maximum shear plane.

$$\frac{\Delta\gamma}{2}\left(k\frac{\sigma_{n,max}}{\sigma_y}+1\right)=\frac{\tau_f'}{G}(2N_f)^{b_\gamma}+\gamma_f'(2N_f)^{c_\gamma} = \left[(v_e+1)\frac{\sigma_f'}{E}(2N_f)^b + (v_p+1)\varepsilon_f'(2N_f)^c\right]\left[k\left(\frac{\sigma_f'}{2\sigma_y}(2N_f)^b\right)+1\right] \quad (17)$$

Lui [65] introduced that is centred on virtual strain energy (VSE), where the stress and strain ranges are multiplied together, as depicted in Equations (18) and (19). In the introduced model, the critical plane is determined based on the value of the maximum normal work, and the VSE quantity is defined as the combined normal work and shear work on the critical plane". This approach is tailored to effectively address the prevalent tensile failure in the material. Conversely, if the model is applied to materials where shear failure is dominant, the VSE calculation is adjusted accordingly, as shown in Equation (19).

For tensile failure:

$$\Delta W=(\Delta\sigma_n\Delta\varepsilon_n)_{max}+(\Delta\tau\Delta\gamma) \quad (18)$$

For shear dominated failure:

$$\Delta W=(\Delta\sigma_n\Delta\varepsilon_n)+(\Delta\tau\Delta\gamma) \quad (19)$$

The effectiveness of the VSE model was reevaluation by Lui and Wang [66]. In their reevaluation, they discovered that the VSE model did well in the prediction of fatigue cracks physical characteristics like crack initiation sites, crack orientations, and fracture mode. However, the model tends to under-predict fatigue life for superimposed compressive mean stress. To address this limitation, a probabilistic formulation of Lui's model was proposed by Núñez and Calvo [67]. This formulation utilizes a perturbation method and statistical moments by using the mean and variance of the random fatigue life variables. When the results of their analyses were compared to the Monte Carlo simulation approach, both results were in good agreement, although, experimental results were not used as the basis for comparing the performance of the model.

Chu [68] introduced a model with parameters similar to those of Lui's, wherein the shear stress and normal work were combined, but the stress range was substituted with maximum stresses in order to incorporate the mean effect, as illustrated in Equation (20); and the plane with the highest fatigue parameter is defined as the critical plane.

$$\Delta W=\left(\sigma_{n,max}\frac{\Delta\varepsilon}{2}+\tau_{n,max}\frac{\Delta\gamma}{2}\right)_{max} \quad (20)$$

In a related study, Glinka and Wang [69] introduced a shear strain energy model that incorporates combined tensile and shear mean stress effects as shown in Equation (21). In the model that was formulated, the critical plane was determined as the plane where the highest shear work was experienced.

$$\Delta W=\frac{\Delta\tau}{2}.\frac{\Delta\gamma}{2}\left(\frac{\sigma_f'}{\sigma_f-\sigma_{n,max}}+\frac{\tau_f'}{\tau_f-\tau_{n,max}}\right) \quad (21)$$

Classical methods, such as those based on equivalent stress, strain, and energy approaches, have been widely employed over time. However, these methods may not adequately account for the complexities of real-world fatigue behaviour, such as multiaxial stress states or uncertainties in material properties and loading conditions. Advancements have been made in the development of multi-axial fatigue criteria, strain-based fatigue models, and probabilistic methods to address these limitations.

4. Improvement in Fatigue Life Estimation Techniques

Accurately predicting fatigue life is essential for ensuring the safety and reliability of mechanical components subjected to cyclic loading. However, estimating fatigue life in real-world situations is a complex task that requires consideration of several key requirements. The following requirements must be fulfilled to improve the accuracy and reliability of fatigue life estimation:

i. **Consideration of loading conditions**: Fatigue life estimation methods should account for the specific loading conditions experienced by the component. These include factors such as non-zero superimposed static stresses, multiaxial stress environments, and stress concentration impacts. The ability to capture the variations and interactions of all stress components throughout the loading period is crucial for accurate fatigue assessment.

ii. **Material-specific characteristics**: Different materials exhibit unique fatigue behaviour, which must be taken into account during fatigue life estimation. The response of materials to cyclic loading can vary significantly, necessitating the development of material-specific fatigue models. Understanding the material's fatigue properties, such as fatigue strength, endurance limit, and fatigue crack growth behaviour, is crucial for accurate estimation.

iii. **Experimental data for model calibration**: To improve the accuracy of fatigue predictions, it is essential to calibrate and validate fatigue assessment methods using experimental data. This data should be obtained from tests conducted according to relevant standard codes, such as ASTM International and ISO, which define specific testing procedures and protocols. By conducting tests according to these standards, researchers and engineers can obtain realistic operating conditions and material responses, leading to more reliable fatigue life estimations.

iv. **Incorporation of uncertainty**: Fatigue life estimation methods should consider uncertainties in material properties and loading conditions. Probabilistic approaches can be employed to account for these uncertainties and provide a more realistic assessment of fatigue life. By considering the statistical distribution of material properties and loading variables, the reliability and confidence of fatigue predictions can be improved.

v. **Computational modeling and simulation**: Advances in computational modelling and simulation techniques have provided valuable insights into fatigue behaviour at the microstructural level. These techniques allow for the analysis of fatigue mechanisms and the evaluation of component performance under various loading conditions. Incorporating these models into fatigue life estimation methods can enhance their accuracy and enable a better understanding of the underlying fatigue processes.

By fulfilling these requirements, fatigue life estimation methods can be improved, thus, leading to more accurate predictions and ultimately enhancing the safety and reliability of mechanical components.The subsequent section outlines recent progress in the techniques employed for predicting multiaxial fatigue life. It also examines the strengths and weaknesses of various techniques/models. The techniques include critical plane analysis, enclosed model approach, integral type model technique, material-structured-based model, stress invariant-based model, statistical assessment model method, and plasticity framework modelling technique.

4.1. Assessment of multiaxial fatigue using the critical plane model

Assessment of multiaxial fatigue using the Critical Plane Model is a valuable method for predicting the fatigue life of materials subjected to complex loading conditions. The Critical Plane Model focuses on identifying the critical planes within the material where fatigue damage accumulation is most likely to occur. By analyzing the stress distribution on these critical planes, one can easily assess the fatigue behaviour of materials more accurately, by considering the combined effects of normal and shear stresses that vary in different directions. This approach is particularly relevant in situations where materials experience multiaxial loading conditions, such as rotating components or structural elements subjected to varying stress states.

The evolution of critical plane methodologies has largely relied on empirical observations of crack initiation and propagation during loading [70]. In general, fatigue cracks are known to initiate and grow on the critical plane [71], thus resulting in more precise life predictions under multiaxial stress/strain conditions as compared to uniaxial fatigue models. Consequently, several critical plane criteria, that are based on various parameters and assumptions have been formulated to describe the mechanisms of fatigue failure of different materials [70]; and these approaches enable the estimation of fatigue life and the determination of crack plane orientations [64].

Critical plane methodologies typically hinge on either the maximum principal strain/stress plane or the maximum shear strain/stress plane to address various types of failures. Consequently, the methodology can be categorised into three groups [58]: stress criteria, strain criteria, and criteria that integrate both stress and strain (also termed energy-based criteria) [44, 49, 59, 60]. However, accurately pinpointing the critical plane is crucial when employing this approach. This is because the critical plane of a component when subjected to a complex stress state typically lies near its stress-strain concentration, as illustrated in Figures 3 and 4.

Critical plane approaches typically rely on either the maximum principal strain/stress plane or the maximum shear strain/stress plane for different types of failures. Consequently, they can be categorized into three groups [72]: stress criteria, strain criteria, and criteria combining both stress and strain (also known as energy-based criteria) [58, 63, 73, 74]. However, the accurate determination of the critical plane is essential when employing this approach. This is because the critical plane of a component subjected to a complex stress state typically lies in the vicinity of its stress-strain concentration, as illustrated in Figures 3 and 4.

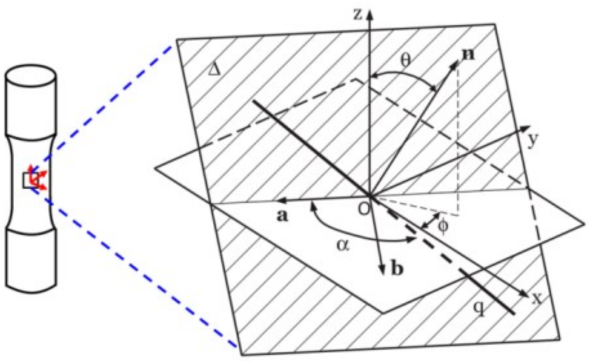

Figure 3. Critical plane determination for multiaxial stress components [70].

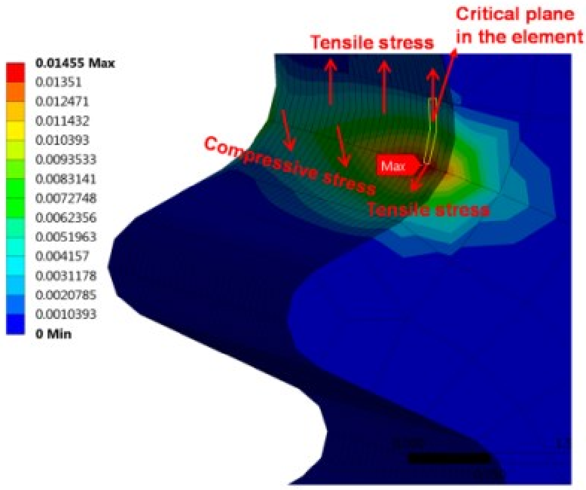

Figure 4. Stress analysis of the element with critical plane [70].

Lee [75] introduced an equivalent stress criterion tailored for intricate multiaxial fatigue, and particularly suitable for addressing out-of-phase bending and torsion. The model performed well when compared with other criteria and with different specimens. Additionally, Furthermore, Lee adapted the Gough ellipse quadrant [76] to integrate the phase difference between loadings, as depicted in Equations (22) and (23); and the acquired experimental data was validated using several materials.

$$\sigma_{eq} = \sigma_a \left[1 - \left(\frac{2b_f \tau_a}{2t\sigma_a}\right)^{\alpha}\right]^{1/\alpha} \tag{22}$$

$$\alpha = 2(1 + \beta \sin \emptyset) \tag{23}$$

Lee also modified Equation (22) and (23) to incorporate the bending mean stress. Bending and torsion tests conducted on structural steel (SM45C) were used to validate the revised equation [75].

$$\sigma_{eq} = \frac{\sigma_a \left[1 + \left(\frac{b_f k}{2t}\right)^{\alpha}\right]^{1/\alpha}}{\left[1 - \left(\frac{\sigma_m}{\sigma_u}\right)^{n}\right]} \tag{24}$$

A critical plane model that assesses the maximum shear stress amplitude alongside the corresponding maximum normal and mean stresses on the identical plane was introduced by Lazzarin and Susmel [77].

Later, they developed a modified Wholer curve method (MWCM), presented in Equations (25) and (26).

$$k_\tau(\rho_{eff}) = a\rho_{eff} + b_f \quad (25)$$

$$\tau_{A,Ref}(\rho_{eff}) = \alpha\rho_{eff} + \beta \quad (26)$$

A simplified approach for applying the theory of critical distance (TCD) was presented by Susmel and Taylor [78] as outlined in Equation (27). The approach used was discovered to be beneficial when evaluating the fatigue behaviour of notched components under torsional fatigue loading, as the approach only requires experimental results from uniaxial tests and linear finite element models (FE) for it to be used.

$$L = \frac{1}{\pi}\left(\frac{\Delta K_{I,th}}{\Delta\sigma_o}\right)^2 \quad (27)$$

Subsequently, a novel method that combines MWCM and TCD was presented by Susmel and Taylor [14], to accurately estimate the fatigue life of components under multiaxial loadings. This method proved to be efficient when it was calibrated to utilize only the necessary speculative information. Moreover, the method has proven effective for estimating fatigue lifetime under variable amplitude uniaxial/multiaxial fatigue loading by directly examining the elastic stress fields causing material damage near stress raisers [5]. Building on the combined MWCM and TCD concept, Susmel [79] developed a technique for critical plane determination. According to this formulation, crack initiation occurs on the material's plane with direction along the variance where the resolved shear stress reaches its maximum, as depicted in Equations (28)-(30). A notable feature of this approach is its swift determination of the critical plane. This is because the time required to identify the global maximum is independent of the length of the load history under examination.

$$Var[\tau_q(t)] = d^T[C]d \quad (28)$$

where,

d = direction cosines, $[C]$ = matrix which consists of variance V_i, covariance $C_{i,j}$ specified as:

$$V_i = Var[\sigma_i(t)], \text{ for } i = x, y, z, xy, xz, \text{ and } yz \quad (29)$$

$$C_{i,j} = CoVar[\sigma_i(t),\ \sigma_j(t)], \text{ for } i = x, y, z. xy, xz, \text{ and } yz \quad (30)$$

Susmel and Taylor [79] reformulated the TCD method to incorporate the estimation of fatigue life for notched components under various uniaxial loadings. They emphasized three types of TCD application: the point method, line method, and area method; and concluded that the simplest method to apply of the three methods identified during load calculation is the line method. Additionally, Additionally, a methodology for fatigue life estimation under variable amplitude loading conditions was presented by Susmel and Tovo [17]. This methodology combines MWCM with the Maximum Variance Method (MVM) for determining the critical plane. They introduced the sum of the critical damage parameter, described in Equations (31) and (32), which varies depending on the degree of multiaxiality and non-proportionality.

$$D_{CR}(\rho_{eff}) = d_1 \cdot \rho_{eff} + d_2 \quad (31)$$

$$\rho_{eff} = \frac{m \cdot \sigma_{n,m} + \sigma_{n,a}}{\tau_a} \quad (32)$$

A method for the evaluation of fatigue in notched components subjected to variable amplitude loading was devised by Susmel and Taylor [5]. This approach integrated MWCM with TCD in the point method, thus, determining the critical plane through MVM. In their conclusion, they stated that this approach offered a high level of precision, thus, enabling the design of actual components against variable amplitude uniaxial or multiaxial loading by analyzing pertinent stress fields obtained directly from conventional linear finite element models.

Using the MVM method, Susmel and Tovo [80] investigated the degree of multiaxiality and non-proportionality in applied loading histories. They found that the path of maximum variance of resolved shear stress accurately estimates the orientation of Stage-I crack paths and the accuracy of the estimation can be increased for non-proportional loading by using the MVM method.

Susmel and Louks [81] validated the efficacy of the combined TCD (used in the point method) and MWCM for the estimation of the fatigue life of components under uniaxial/multiaxial loading by utilizing the results obtained from finite element analysis software. Their results demonstrated a high degree of accuracy in the computed fatigue life.

A fatigue model with the concept based on the deviation of the critical plane was introduced by Mahadevan and Liu [82], as expressed in Equation (33). With this model, fractures are first identified and, thereafter, the critical plane is determined at a certain deviation from the plane of the fracture surface. This idea stemmed from the observation that the initiation of crack occurs on one plane while the propagation occurs in another, along a different orientation plane. The comparison of the model's data with experimental results for constant amplitude loading revealed good agreement between both data. Expanding on this, Mahadevan and Lui [83] extended the model to incorporate anisotropic and composite materials. However, they were unable to test the proposed model due to a lack of fatigue damage experimental for anisotropic materials in the open literature. Nonetheless, they concluded that validation of the model could be feasible in future studies once the necessary data become available.

$$f_{Nf} = \frac{1}{\beta}\sqrt{\sigma_{a,c}\left[\left(\mu N_f \frac{\sigma_{m,c}}{f_{Nf}} + 1\right)\right]^2 + \left(\frac{f_{N_f}}{t_{N_f}}\right)^2 \left(\tau_{a,c}\right)^2 + \sigma_{a,c}^{H\ 2}} \quad (33)$$

A non-linear fatigue damage function that adopts the critical plane concept for computing fatigue damage was introduced by Ninic and Stark [84]. Their expression, as depicted in Equation (34), is termed by Papuga [21] as the quadratic critical plane formula, and it is also identified as the plane where the damage function reaches its maximum value. Emphasis is placed on the sensitivity factor of the normal stress for precise predictions using this fatigue damage function. Additionally, the significance of the endurance strength ratio in multiaxial fatigue analysis is highlighted.

$$D(l,m,n) = \left[\left(\frac{\tau_a(l,m,n)}{T_e}\right)^2 + k'\left(\frac{\sigma_{eq}(l,m,n)}{S_e}\right)^2\right]^{1/2} \quad (34)$$

With a focus on the impact of shear stress in relation to normal stress, a damage parameter with two similar criteria was proposed by Papuga and Ruzicka [85]. The expression for the identification of the critical plane criteria for the criteria is expressed in Equations (35)-(37). The objective of the study was to assess the effectiveness of incorporating or reducing the load effect. Ultimately, both methodologies produced similar outcomes based on the dataset presented.

$$\sqrt{a_c c_a^2 + b_c \cdot \left(N_a + \frac{t_{-1}}{f_0} \cdot N_m\right)} \leq f_{-1} \quad (35)$$

where,

$$k < \sqrt{\frac{4}{3}}, \quad a_c = \frac{k^2}{2} + \frac{\sqrt{k^4 - k^2}}{2}, \quad b_c = f_{-1} \quad (36)$$

$$k \geq \sqrt{\frac{4}{3}}, \quad a_c = \left(\frac{4k^2}{4 + k^2}\right)^2, \quad b_c = \frac{8f_{-1}k^2(4 - k^2)}{(4 + k^2)^2} \quad (37)$$

By incorporating an initial flaw size as illustrated in Equations (38) and (39), a critical plane model was introduced by Lu and Lui [86]. In the model, they employed the maximum normal stress plane along with the stress intensity coefficients ratio of mode II to mode I, which correspond to the specific crack growth rate in order to determine the critical plane.

$$N = \int_{ai}^{ac} \frac{1}{C\left[\Delta K_{eq} - \Delta K_{th}\right]^m} da \quad (38)$$

$$K_{mixed,eq} = \frac{1}{B}\sqrt{(k_1)^2 + \left(\frac{k_2}{s}\right)^2 + A(k^H)^2} \quad (39)$$

Another critical model that utilizes a damage parameter as expressed in Equation (40) to identify critical planes characterized by maximum shear strain and high displacement in the normal strain path was introduced by Shang and Sun [87]. The study illustrated a significant correlation between the proposed parameter and the multiaxial fatigue life of materials, especially under low-cycle loading conditions.

$$\left(\varepsilon_n^* + \frac{v_{eff}(2 - v_{eff})}{(1 + v_{eff})^2}\frac{\Delta\gamma_{max}}{2}^2\right)^{1/2} = \frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c \quad (40)$$

Li and Sun [88] as well as Li and Zhang [89], introduced a straightforward critical plane approach for evaluating the fatigue life of metallic materials subjected to proportional and non-proportional loading conditions, as outlined in Equation (41). In this model, the maximum shear strain range ($\Delta\gamma_{max}$) and normal stress ($\sigma_{n,max}$); and normal strain range ($\Delta\varepsilon_n$) are specified on the maximum strain range plane.

$$\frac{\Delta\varepsilon_{eq}^*}{2} = \frac{\Delta\gamma_{max}}{2} + \left(\frac{\sigma_{n,max}}{\sigma_y} + 1\right)\frac{\Delta\varepsilon_n}{2} = \left[\frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c\right]\left(\frac{\sigma_f'}{\sigma_y}(2N_f)^b + 1\right) \quad (41)$$

Two distinct forms of the generalized strain energy damage parameters that are associated with the maximum fatigue damage plane as outlined in Equation (42) were proposed by Ince and Glinka [30]. Additionally, they expressed the generalized amplitude strain energy damage parameter as depicted in Equation (43).

$$W_{gen}^* = \left(\tau_{max}\frac{\Delta\gamma^e}{2} + \frac{\Delta\tau}{2}\frac{\Delta\gamma^p}{2} + \sigma_{n,max}\frac{\Delta\varepsilon_n^e}{2} + \frac{\Delta\sigma_n}{2}\frac{\varepsilon_n^p}{2}\right)_{max} = f(N_f) \quad (42)$$

$$\frac{\Delta\varepsilon_{gen}^*}{2} = \left(\frac{\tau_{max}}{\Delta\tau/2}\frac{\Delta\gamma^e}{2} + \frac{\Delta\gamma^p}{2} + \frac{\sigma_{n,max}}{\Delta\sigma_n/2}\frac{\Delta\varepsilon_n^e}{2} + \frac{\Delta\varepsilon_n^p}{2}\right)_{max} = f(2N_f) \quad (43)$$

Using the critical plane approach as the basis, a high-cycle multiaxial fatigue criterion was presented by Carpinteri-Spagnoli [90]. While incorporating the material's fatigue properties through the angle of rotation, the orientation of the critical plane aligns with the average direction of the principal stress axis. Estimating multiaxial fatigue strength involves computing an equivalent stress amplitude on the critical plane. Furthermore, the model's formulation relies on the ratio between the fatigue limit under fully reversed shear stress and that under fully reversed normal stress. This approach is versatile and thus is suitable for metals exhibiting fatigue behaviour ranging from mild to tough.

Anes et al. [91] proposed a Stress Scale Factor, SSF polynomial function that is linked to the Stress Amplitude Ratio, SAR between shear and normal components. The function proposed is based on the tension-torsion history, which combines the normal stress and the shear stress amplitudes applied on the cross-section of the specimen. Demonstrated success in multiaxial fatigue life prediction has been observed under both variable amplitude loading [92] and constant amplitude loading [93]. Its versatility extends to different loading scenarios, encompassing both constant [94, 95] and variable amplitude loading [96].

Applying the critical plane approach involves assessing damage on the plane where it is maximized, typically by employing a constant SSF (Stress Scale Factor) value corresponding to the material in question. In cases involving brittle materials, Goodman's model coupled with the maximum principal stress proves to be more suitable. In the realm of uniaxial fatigue modelling, structures' fatigue life is commonly predicted by accounting for non-zero mean stress under combined thermal acoustic loadings [97]. When metal structures face such loading conditions, they encounter multiaxial fatigue issues, thus, complicating the cumulative assessment by introducing significant errors in the outcome of the uniaxial model [98]. The critical plane model provides a physical explanation of the multiaxial fatigue damage mechanism, by allowing for fatigue life prediction while accounting for the influences of mean stress.

Furthermore, Ge et al. [99] introduced a novel critical plane model that predicts metallic structures' fatigue life under combined thermal and acoustic loading. In order to accommodate induced mean stresses resulting from temperature loading, they further proposed a new model that relies on shear strain. Experimental data from literature and tests conducted on four different materials under strain paths, with zero and non-zero mean stress, were used to validate the model, and the results of the model were in good agreement with those obtained experimentally. When the model was applied to predicting the fatigue life of metal structures subjected to thermal-acoustic loading, and the results obtained were compared with the uniaxial Goodman model, the proposed model appeared to be conservative, as it significantly reduces the fatigue life due to thermal loading.

4.2. Fatigue analysis based on enclosed surface model

Fatigue analysis using the Enclosed Surface Model is also another valuable method for predicting material fatigue life under cyclic loading. By examining the stress distribution enclosed by the hysteresis loop on the stress-strain curve, this model offers insights into the material's response to cyclic loading and potential fatigue failure. The technique provides a visual representation of stress-strain behaviour and captures energy dissipation over loading cycles, thus, aiding in the understanding of fatigue resistance and durability of components. This method is beneficial when assessing materials exposed to varying stress levels and mean stresses, as the method offers a systematic approach for predicting fatigue life, and identifying critical areas prone to fatigue damage.

Mamiya and Goncalves [100] introduced a model for determining the multiaxial high-cycle fatigue endurance criterion under sinusoidal iso-frequency loading, encompassing both in-phase and out-of-phase conditions. Their model relies on the principle of the minimum circumscribed ellipse within Ilyushin's deviatoric space, as depicted in Equation (44). The results obtained from this study were compared with other models and it was discovered that the model is conservative in the prediction of the useful life of components when mean stress is used.

$$\sum_{i=1}^{5} a_i^2 + k\sigma_{p\,max} \leq \delta \quad (44)$$

By employing the prismatic hulls along the principal axis directions, the enclosing path method was introduced by Leila and Emmanuel [101], as outlined in Equation (45). The technique captures the equivalent stress parameter for various loads that lead to the same stress parameter value.

$$\sqrt{J_{2,a}} = \sqrt{R_1^2 + R_2^2 + R_3^2 + R_4^2 + R_5^2} \quad (45)$$

The maximum rectangular hull method was employed by Araujo and Dantas [102] to describe the equivalent shear stress amplitude as detailed in Equations (46) and (47). This method entails maximizing the size of a rectangle by fitting it onto a complex loading path. Through this approach, both proportional and non-proportional loading can be discerned, thus, leading to improved predictions as compared to the minimum rectangular hull method when both sets of data were compared with experimental results.

$$\tau_a^{max} = \max\{\tau_a(\emptyset, \theta)\} \quad (46)$$

where,

$$\tau_a = \max\sqrt{a_1^2(\varphi) + a_2^2(\varphi)} \quad (47)$$

In another study, Mamiya and Castro [103] presented a fatigue life estimation model that operates on a piecewise rule. This model incorporates two surfaces: the first one combines the deviatoric stress amplitude with the maximum hydrostatic stress, following an exponential function outlined in Equation (48), while the second surface only uses the deviatoric stress amplitude, as described in Equation (49) when estimating fatigue life. Conversely, in situations with low magnitudes of hydrostatic stress, only the second surface that considers the deviatoric stress amplitude is used for the estimation. While this model's performance aligns with that of other models proposed in the study, it shows superior performance, particularly in scenarios involving mean stress.

$$\tau_a + \sigma_{H\,max} = \alpha N_f^{\beta} \ \text{if}\ \sigma_{H\,max} \geq \alpha\left(\frac{\tau_a}{\gamma}\right)^{\beta/\delta} - \tau_a \quad (48)$$

$$\tau_a = \gamma N_f^{\delta} \quad (49)$$

An enclosed surface model that incorporates the modified Wang-Brown rainflow method was introduced by Meggiolaro and de Castro [33, 104]. This model aimed to address the discrepancies observed in previous enclosed surface methods. Building upon the minimum hull method, their approach considered portions of the component with more than one cycle and treated them separately to avoid information loss. Prior methods often failed to account for the actual loading path, as they focused solely on convex hulls, and this potentially led to underestimated damage.

The limitations associated with the original Wang-Brown model were evident when the model was used to handle load histories with multiple cycles that are counted by the rainflow methods before the application of the moment of inertia. To rectify this, Meggiolaro and de Castro proposed two enhancements: modifying the counting starting point to refine the algorithm's implementation and formulating a 5-dimensional (5-D) Euclidean stress space using the deviatoric stress tensor. Additionally, Marsh et al. [105] conducted a review of the residual data point post-processing technique, which tackles the constraints of traditional rainflow counting methods when confronted with gradual but continuous changes in mean stress.

4.3. Fatigue analysis based on integral type models

Fatigue analysis using Integral Type Models is an advanced method for predicting material fatigue life under cyclic loading. By integrating the stress history of the material over time, these models provide a detailed understanding of fatigue behaviour, considering cumulative stress effects and enabling accurate predictions of component life. The models assess how different stress parameters impact fatigue performance by analyzing stress variations over time. Integral-type models capture transient material behaviour under cyclic loading, thus, offering insights into fatigue damage accumulation and informing decisions on design optimization and maintenance practices for enhanced durability of components. By combining experimental data, mathematical formulations, and simulations, these models identify critical stress states, thus, facilitating effective mitigation strategies that help to extend component lifespan.

An integral type fatigue criterion that integrates all its components across all planes at the point of interest was introduced by Papadopoulos [106]. Additionally, the model was devised such that it incorporates an extra integration of resolved shear stress over the shear plane, as outlined in Equations (50) and (51). Although this model is effective for hard metals [107], its computational demands are high, thus, limiting its practical application in commercial fatigue solvers [21].

$$\sqrt{\langle T_a^2 \rangle} + \alpha\sigma_{h,max} \le \beta \quad (50)$$

$$\sqrt{\langle T_a^2 \rangle} = \sqrt{5}\sqrt{\frac{1}{8\pi^2}\int_{\theta=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{x=0}^{2\pi}\left(T_a(\varphi,\theta,x)\right)^2 dx \sin\theta \, d\theta d} \quad (51)$$

In a research conducted by Lasserre and Palin-Luc [108], they introduced an energy density model rooted in the idea of volumetric strain energy density distribution around the critical point linked to fatigue failure. The prediction of the model, depicted in Equation (52), exhibited good agreement with experimental data obtained from both uniaxial and multiaxial tests conducted on smooth cylinder specimens.

$$\omega_a^D(C_i, load) = \frac{1}{V^*(C_i)}\int\int\int[_{V^*(C_i)} W_a(x,y,z,load)] - W_a^*(C_{i,load})]dv \quad (52)$$

The damaged part of the volumetric density model was improved by Palin-Luc and Banvillet [109] to address the deficiency related to associated with the fully reversed sinusoidal loading in the original model [108]. The model was reformulated by Saintier and Palin-luc [110] as expressed in Equation (53). This reformulation also enhanced the criterion for incremental fatigue life prediction under both proportional multiaxial and non-proportional multiaxial, variable amplitude loadings. Within the experimental cases, the proposed model gave an accurate fatigue life estimate. However, a thorough study with different materials was suggested in order to further give credence to the model.

$$W_{geqdam}(M) = \langle W_{geq}(M) - \sum_{i=1}^{6} P_l(M)\alpha_{M,1}W_g^* \rangle \quad (53)$$

The fatigue life estimation criterion presented by Zenner and Simburger [111] incorporates an integral type methodology and shear stress intensity, as depicted in Equation (54). Although this model provided accurate estimations for complex periodic loadings, it proved unsuitable for the low-cycle fatigue regime. This limitation arises because the specification of strain-based parameters is necessary to address plastic deformation, rather than stress quantities [111].

$$\sigma_{equ,\alpha} = \left\{\frac{15}{8\pi}\int_{\gamma=0}^{\pi}\int_{\varphi=0}^{2\pi}\left[\alpha\tau_{\gamma\varphi\alpha}^{2}\left(m\tau_{\gamma\varphi m}^{2}+1\right) + b\sigma_{\gamma\varphi\alpha}^{2}\left(n\sigma_{\gamma\varphi m} + 1\right)\right]\sin\gamma d\gamma d\varphi\right\}^{1/2} \quad (54)$$

Two criteria that focus on the influence of shear stress in relation to normal stress were introduced by Papuga and Ruzicka [85]. These criteria utilize a similar damage parameter, and they both integrate the fatigue parameter over all planes, as expressed in Equations (55) and (56). The objective of the model was to assess the efficiency of two approaches: integrating the load effect versus minimizing it. Ultimately, both methods yielded very similar results.

$$\sqrt{\frac{1}{4}\int_{\varphi}\int_{\vartheta}\left[a_1\cdot C_a^2 + b_1\cdot\left(N_a + \frac{f_{-1}}{t_{-1}}N_m\right)\right]\sin\vartheta d\vartheta d\varphi_{-1}} \leq f \quad (55)$$

where,

$$a_1 = \frac{5}{2}k^2,\ \ b_I = f_{-1}(3-k^2) \quad (56)$$

4.4. Fatigue assessment using material structure models

Fatigue assessment using Material Structure Models is an important approach used for predicting material fatigue performance by analyzing the inherent structural properties influencing fatigue behaviour. These models consider the microstructural aspects like grain boundaries, crystallographic orientation, and defects to ascertain how these features impact fatigue resistance and failure susceptibility. By understanding deformation mechanisms and crack initiation paths, engineers can optimize material processing, composition, and treatments to enhance fatigue life. Material Structure Models offer precise predictions of fatigue crack growth, considering the material's internal characteristics to tailor design solutions, improve fatigue properties, and advance engineering applications for enhanced durability. The use of this methodology allows for a comprehensive evaluation of material behaviour under varying loading conditions, by providing insights into fatigue damage mechanisms and critical regions susceptible to crack formation in components.

A multi-scale damage criterion designed to predict the initiation cracks as expressed in Equations (57) and (8) was proposed by Luo and Chattopadhyay [112]. This approach employs optimization theory to determine the local damage state before progressing to the grain level. It also allows for the assessment of damage within a meso-representative volume element (RVE) comprising multiple grains. Experimental results at a structural hotspot align with the predicted failure outcomes of the RVE. Additionally, the criterion accommodates potential directions of crack growth. However, due to its complexity and strong reliance on grain structure, this method is less suitable for designs where the component's life is represented by its average performance and behaviour.

$$dD^{(\alpha)} = \left(\frac{\sigma_{mr}}{\sigma_o} - 1\right)^{m'}\left(\frac{\sigma_n^{(\alpha)}}{\sigma_f} + 1\right)dY^{\alpha'} \quad (57)$$

$$dY^{(\alpha')} = \delta'\sigma_n^{(\alpha')}(d\varepsilon^p)^{\alpha'} + \frac{1-\delta'}{2}\sigma_s^{(\alpha')}(d\gamma^p)^{(\alpha')} \quad (58)$$

4.5. Stress invariant-based fatigue assessment models

Stress Invariant-Based Fatigue Assessment Models are a robust methodology for predicting material fatigue life under cyclic loading conditions by focusing on stress invariants that remain constant during deformation, as this approach provides a stable basis for the fatigue assessment. Also, these models enable a comprehensive understanding of material response to cyclic loading, through the identification of potential fatigue failure mechanisms and critical stress areas. By considering multiaxial stress states and interactions, the models offer accurate fatigue behaviour analysis as they capture the influence of stress states on material deformation and damage accumulation, by incorporating normal and shear stresses while enhancing the predictive accuracy.

A customized model (void crack nucleation) suitable for ductile metals containing second phases was presented by Horstemeyer and Gokhale [113], as outlined in Equation (59). This model leverages factors such as the material's fracture toughness, the second phase volume fraction of the material, the length scale parameter, the state of stresses and the level of strain. Following this, Lugo and Jordon [114] proposed assessing the model's effectiveness through an acoustic emission-based method.

$$\epsilon(t) = C_{coeff}\exp\left(\frac{\varepsilon(t)d^{1/2}}{K_{IC}f^{1/3}}\left\{a'\left[\frac{4}{27}+\frac{J_3^2}{J_2^3}\right]+b'\frac{J_3}{J_2^{3/2}} + C'\left\|\frac{I_1}{\sqrt{J_2}}\right\|\right\}\right) \quad (59)$$

In another study, Vu and Halm [115] proposed a stress invariant-based fatigue life estimation criterion that introduces a quantity to account for shear stress and phase shift effects, as illustrated in Equations (60) – (62). The model exhibited strong performance within the evaluated experimental data set, thus, prompting recommendations for more extensive studies to further validate its reliability.

$$f = \sqrt{\gamma_1 J_2'(t)^2 + \gamma_2 J_{2,mean}^2 + \gamma_3 I_f(I_{1,a}, I_{1,m})} \leq \beta \quad (60)$$

For low-strength metals,

$$I_f(I_{1,a}, I_{1,m}) = I_{1,a} + I_{1,m} \quad (61)$$

For high-strength metals,

$$I_f(I_{1,a}, I_{1,m}) = I_{1,a} + \frac{f_{-1}}{t_{-1}} I_{1,m} \quad (62)$$

4.6. Statistical approach for fatigue assessment

The Statistical Approach for Fatigue Assessment utilizes statistical analysis techniques to predict material fatigue life under cyclic loading by analyzing data on material properties, loading conditions, and fatigue test results to develop models for estimating the likelihood of fatigue failure. This approach considers factors like material heterogeneity and loading variability to provide a realistic estimation of component life. The technique employs probabilistic modelling and sensitivity analyses to identify influential factors impacting fatigue life, thus, enabling engineers to optimize designs and maintenance strategies. By addressing uncertainty and variability, this method enhances reliability in fatigue life predictions, as it offers valuable insights for decision-making, design optimization, and improving the durability and performance of engineering components in the long term.

Leveraging the Weibull statistical framework for assessing stress-life relationships in a probabilistic manner, Pinto and De Jesus [116] introduced a model that is based on the Weibull regression. This model gives an analytical probabilistic representation of the entire strain-life field, thus, providing quantile curves for high and low regions of the fatigue. Unlike conventional approaches, this model directly manages total strain without requiring the segregation of elastic and plastic strain constituents. This feature sets the model apart, allowing it to handle runouts and support probabilistic forecasts of lifetime through damage accumulation techniques [2].

4.7. Plasticity-based models for fatigue assessment

Plasticity-Based Models for Fatigue Assessment are advanced tools that predict how materials respond to cyclic loading by considering plastic deformation effects. These models focus on the material's inelastic behaviour, including plastic strains and cyclic hardening, to understand fatigue damage mechanisms accurately. By analyzing the interaction between plasticity and stress states, the Plasticity-Based Models provide insights into fatigue crack initiation and growth, thus, enhancing the prediction of failure. The model enables engineers to evaluate material fatigue resistance effectively and optimize design parameters to improve durability. By exploring the relationship between plastic deformation and fatigue damage, these models offer a comprehensive understanding of material fatigue behaviour, thus, aiding in the development of strategies to enhance component performance and prevent premature failure. The use of plasticity-based models contributes to advancing fatigue assessment techniques and optimizing the fatigue resistance of engineering structures.

A model capable of assessing fatigue life under multiaxial loading conditions as depicted in Equation (63) was devised by Emuakpor and George [117]. In their model, the nonlinear plastic stress-strain relationships were integrated within distortion theory during cyclic loading. Their approach employs a criterion founded on the concept that the quantity of physical damage culminating in failure is equivalent to the accumulated strain energy experienced during a monotonic loading, in addition to the strain energy experienced during fatigue failure.

Based on extensive experimental data, Chaussumier and Mabru [118] presented a model for predicting fatigue life. Their approach integrates factors like multi-site crack initiation, coalescence among neighbouring cracks, and different stages of short-crack growth, and long-crack propagation. The model was crafted through experimental investigations on surfaces prepared with topographical pickling, enabling accurate identification and analysis of pit dimensions.

Meanwhile, Khandewal and El-Tawil [119] introduced a mechanical-based damage model tailored for simulating ductile fracture behaviour in structural steel. This model operates within a plasticity framework based on the principles of principal effective stress and equivalent strain, as outlined in Equations (64)-(66). Implementation of this model in finite element software necessitates calibration of model parameters, which are inherently dependent on mesh characteristics. Consequently, recalibration is required when employing different materials or changing the mesh sizes.

$$\sigma_E = \frac{\sigma_C}{2}\ln\left(\sum_{p=1}^{3} e^{2\sigma P/\sigma C}\right) \tag{63}$$

$$\dot{\alpha}^d = a_0 T_1(\alpha^P)\dot{\alpha}^P + a_1 T T_1(\alpha^P)\dot{\alpha}^P + a_2 \exp(a_3\alpha^P) T_2(\alpha^P)\dot{\alpha}^P \tag{64}$$

$$\begin{cases} T_1(\alpha^P, \varepsilon_n, \Delta\varepsilon) = 0 & \text{if } \alpha^P < \varepsilon_n \\ T_1(\alpha^P, \varepsilon_n, \Delta\varepsilon) = \dfrac{(\alpha^P - \varepsilon_n)^2}{\Delta\varepsilon^2}\left[3 - \dfrac{2(\alpha^P - \varepsilon_n)}{\Delta\varepsilon}\right] & \text{if } \varepsilon_n < \alpha^P < \varepsilon_n + \Delta\varepsilon \\ \text{and } T_1(\alpha^P, \varepsilon_n, \Delta\varepsilon) = 1 & \text{if } \alpha^P \geq \varepsilon_n + \Delta\varepsilon \end{cases} \tag{65}$$

$$\begin{cases} T_2(\alpha^P, \varepsilon_c, \Delta\varepsilon) = 0 & \text{if } \alpha^P < \varepsilon_c \\ T_2(\alpha^P, \varepsilon_c, \Delta\varepsilon) = \dfrac{(\alpha^P - \varepsilon_c)^2}{\Delta\varepsilon^2}\left[3 - \dfrac{2(\alpha^P - \varepsilon_c)}{\Delta\varepsilon}\right] & \text{if } \varepsilon_c < \alpha^P < \varepsilon_c + \Delta\varepsilon \\ \text{and } T_2(\alpha^P, \varepsilon_n, \Delta\varepsilon) = 1 & \text{if } \alpha^P \geq \varepsilon_n + \Delta\varepsilon \end{cases} \tag{66}$$

4.8. Fatigue assessment utilizing optimization algorithms

Fatigue Assessment Utilizing Optimization Algorithms is an advanced methodology that optimizes design parameters, material properties, and loading conditions to predict fatigue life accurately by incorporating optimization techniques like genetic algorithms and simulated annealing. The approach iteratively refines parameters to enhance component durability and reliability, efficiently minimizing fatigue damage and extending the operational lifespan. By exploring various design possibilities and considering trade-offs, engineers can navigate complex design spaces, increase fatigue performance, and address uncertainties, leading to robust predictions of fatigue life and failure probabilities. With the integration of optimization algorithms, it is possible to obtain tailored solutions for specific engineering applications, thus, enabling engineers to optimize performance, reduce premature failure risks, and enhance component longevity through a systematic and data-driven approach to fatigue assessment.

The methodology proposed by Bukkapantnam and Sadananda [120], offers a systematic approach to analysing the dynamics of crack growth in materials under alternating service conditions without extensive experimentations. Their approach is rooted in a unified framework that accounts for various factors such as load ratio effects, microscopic cracks, dislocation shielding, under-loading, overloading, and surface cracks. By integrating these observations, they derived the structural foundation of the model and utilized a genetic algorithm (GA) to parameterize it. The GA facilitated the reconciliation of complex, yet unknown, physical relationships with empirical data, making the model predictive and analytically tractable. This method offers advantages over purely empirical approaches like neural networks, as it incorporates physically motivated mathematical structures. When applied, the fatigue crack growth model derived from this framework exhibited approximately a 12% error rate in predicting crack growth rates. Nevertheless, this framework shows promise in reducing the necessity for extensive experimentation in fatigue crack growth analysis, thus, offering a promising direction for future research [121].

In addition, Liu [121] introduced a computational procedure centred on simulation to forecast multiaxial fatigue life. This approach merges the “Monte Carlo simulation technique with stochastic process theory and a response surface method”. By doing so, it accommodates the randomness inherent in material properties, applied loading conditions, and geometric factors. The methodology operates under the assumption that failure occurs either when accumulated damage surpasses a predefined threshold or when a crack attains a critical size. It evaluates time-dependent failure probabilities and computes probabilistic life distributions using Monte Carlo simulation. By considering multiple sources of variation, the method yields empirical formulas for the damage accumulation process through the combination of surface response methods and experimental design. Results obtained from field data demonstrated excellent agreement with numerically predicted outcomes.

A comparative study on fatigue life prediction for composite materials, employing a Genetic Algorithm (GA) in conjunction with conventional methods was conducted by Vassilopoulos and Georgopoulos [122]. Their findings demonstrated that the GA approach yielded more accurate fatigue life results compared to traditional methods. Unlike the conventional techniques, the GA model does not rely on specific assumptions about the data distribution; instead, it allows the data to assume a particular statistical distribution, with stress cycle curves following a power curve equation.

One notable advantage of the GA model is its independence from material type. By correlating input parameters with output results, the GA model establishes a comprehensive relationship applicable to various materials, given the availability of requisite data. Based on these findings, Vassilopoulos and Georgopoulos suggested that future research in GA-based fatigue modelling should explore more intricate genetic programming configurations. Additionally, they suggested integrating multiple input variables like off-axis angle, maximum stress, stress amplitude, stress ratio, and the associated number of cycles to failure for each dataset. These improvements have the potential to enhance the predictive precision and versatility of GA-based fatigue models.

A material model to describe the elastoplastic behaviour of materials under cyclic loading was proposed by Franulovic and Basan [123]. Due to the highly non-linear nature of low cycle fatigue behaviour described by the constitutive model, identifying parameters for the model requires complex numerical procedures, like Genetic Algorithms (GA), which allows for the use of a stochastic search methods for approximating solutions to complex problems. Material parameters obtained through calculation are validated by comparing their response (from numerical solutions) with experimental data. The finite element method is commonly used to simulate the material's response in GA parameter identification, thus, the method proves to be an excellent choice for providing fast and reliably convergent results with suitable genetic operators if the GA calculation procedure is used.

A procedure for structural optimization which merges geometrical modelling, optimization and structural analysis into a unified automated computer-aided design framework was introduced by Krishnapillai and Jones [124]. The demonstration of the procedure using fatigue-based optimization technique in conjunction with GA for lightweight structure showed that the procedure is a robust methodology which can be applied to structures with multiple optimum peaks and complex configurations.

In an attempt to quantify the accumulation of damage in materials from their initial to the final stage when subjected to arbitrary multiaxial loading conditions, a “continuum mechanics-based endurance function” was proposed by Brighenti and Carpinteri [13]. To effectively estimate the values of the model's multiple parameters, particularly in assessing the impact of intricate stress histories on fatigue life, they utilized genetic algorithms for numerical evaluation. Contrary to the traditional methods, genetic algorithms (GAs) provide benefits in tackling problems marked by several minima and non-convexity, while circumventing numerical instabilities and the potential oversight of the global optimum. GAs function through the use of straightforward principles such as random number generation, switching, choice, and combination, thus, enabling them to address any objective function without the need for specific plane or cycle counting algorithms. The approach assesses the accrued damage over the loading phases, under the assumption that fatigue life primarily hinges on crack initiation, and the results from the use of the technique showed satisfactory alignment with experimental results especially when employing the sophisticated endurance function.

The endurance function model was examined by Kamal et al [125], and they proposed that the model can be simplified if the number of parameters calibrated is reduced. By doing this, they developed a methodology that utilizes the stress results from finite element analysis to estimate fatigue life, thus, offering a practical approach for assessing fatigue in structures subjected to multiple variable loadings. Roux and Lorang [126] introduced a method for defining Equivalent Fatigue Load (EFL) from in-service load measurements, thereby, facilitating full-scale structural tests for experimental validation. They also proposed a mathematical method that employs Genetic Algorithms (GA) to accurately compute EFL for the entire structure, hence, enabling benchwork validation tests and optimization of structural geometry during the design phase.

Lotfi and Beiss [127] applied Artificial Neural Networks (ANN) to estimate the impacts of different powder metallurgical processing/fabricating parameters on the endurance limit of powdered steel samples. Leveraging preexisting data from published experimental studies, they integrated genetic algorithms (GA) to refine experimental parameters within practical constraints, with the aim of attaining the targeted fatigue resistance. This combined approach involving GA proved to be efficient and cost-effective, as it helps in facilitating the identification of optimal material compositions and processing conditions for bolstering fatigue durability.

A stress-based model technique that takes into consideration the effect of the value of mean stress on constructional materials fatigue strength was introduced by Niesłony and Böhm [128]. Their fatigue prediction model, paired with the locally devised optimization algorithm, guarantees the utmost accuracy in assessing fatigue life. Central to their approach is the assumption of adjusting fatigue strength amplitudes according to two extreme conditions: tensile and compressive, with a stress ratio (R) of 1. However, a constraint emerges when addressing average stress values under unilateral tension (R=0).

Inspired by the foraging behaviour of ants, Dorigo et al. [129] developed ant-stigmergy optimization methods to tackle the complex optimization problems associated with independent continuous variables. In an attempt to address these complexities, Korosec et al [130] introduced the Differential ant-stigmergy algorithm (DASA) as a variant of ant-stigmergy algorithms.

A method for computing strain cycle ($\varepsilon - N$) curves and how they are scattered was proposed by Klemenc and Fajdga [131, 132]. They established five parameters, wherein four are Coffin-Manson equations pertaining to the scale parameter of the Weibull distribution, while the fifth characterizes the shape parameter. Employing Differential ant-stigmergy algorithm (DASA) and Genetic Algorithms (GA), they efficiently estimated these parameters from a small set of experimental data points, with GA proving to be faster in producing results.

In general, while optimization algorithms like artificial neural networks and genetic algorithms show promising potential for fatigue life estimation, there are challenges and limitations to consider. One of such challenges is the requirement for a large amount of quality training data to effectively train the algorithms. Obtaining such data can be time-consuming and costly, especially for complex loading scenarios and diverse materials. Another challenge is the interpretability of the results obtained from optimization algorithms. These algorithms often work as black boxes, thus, making it difficult to understand the underlying mechanisms or factors contributing to the fatigue life estimations. Additionally, the performance of optimization algorithms can be influenced by the quality and representation of the input features and the complexity of the fatigue phenomena being modelled. Thus, careful selection and representation of input parameters are crucial to achieve accurate and reliable results. Despite these challenges, optimization algorithms offer promising avenues for enhancing the efficiency and accuracy of fatigue life estimation, and further research and development in this area can help overcome these limitations.

5. Conclusion

This review presents a spectrum of fatigue life estimation models, ranging from classical approaches (stress, strain, and energy-based) to more advanced ones (critical plane, enclosed surface, integral type models, etc.). It emphasizes that fatigue crack nucleation and propagation typically occur on critical planes, making the critical plane more accurate for the prediction of multiaxial stress/strain states as compared to uniaxial fatigue models. The Stress Scale Factor (SSF) approach is also discussed as an alternative to critical plane-based methods because of its efficiency under both constant and variable amplitude loading.

The Volume Strain Energy (VSE) model is highlighted for its capability to predict fatigue crack characteristics such as initiation site, orientation, and fracture mode. However, it tends to under-predict fatigue life for superimposed compressive mean stress. The review shows how optimization algorithms parameterized using Genetic Algorithms (GA) facilitate crack growth identification under cyclic loading conditions and reconcile physical relationships with empirical observations. Combining conventional methods with GA tools for fatigue life estimation yields more accurate results than traditional methods, given the former's material independence. Finite Element (FE) methods employed to simulate material responses in GA parameters offer fast and reliable convergence. Additionally, GA outperforms the Differential Ant-Stigmergy Algorithm (DASA) in result generation speed. The integration of Artificial Neural Network (ANN) models with GA is lauded as a cost-effective and powerful optimization tool for selecting optimal material compositions and operating conditions to meet desired fatigue strength requirements.Despite these advancements, the review acknowledges the limitations of existing models, thus, emphasizing the need for further research to develop a universal model capable of reliably and accurately estimating fatigue life across various materials and loading conditions. Considering the limitations of existing fatigue estimation models, future research can focus on several areas to contribute to the development of a widely accepted model. First, there is a need for further understanding and characterization of fatigue behaviour in diverse materials and loading conditions. This can be achieved through experimental studies, advanced testing techniques, and comprehensive data collection. Additionally, the integration of advanced computational methods, such as machine learning and data-driven approaches, can enhance the accuracy and efficiency of fatigue life estimation. Moreover, collaborative efforts between researchers, industry experts, and standardization organizations can lead to the development of standardized guidelines and protocols for fatigue assessment that encompass a wide range of materials and loading scenarios. By addressing these research directions, we can pave the way for the development of a more universally applicable model for fatigue life estimation.

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Competing Interest Statement

The authors declare no competing interests.

Data availability

Data will be made available upon request.

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References

[1] W. D. Callister Jr and D. G. Rethwisch, *Fundamentals of materials science and engineering: an integrated approach*. John Wiley & Sons, 2020.

[2] M. Kamal and M. Rahman, "Advances in fatigue life modeling: A review," *Renewable and Sustainable Energy Reviews,* vol. 82, pp. 940-949, 2018.

[3] M. D. Sangid, "The physics of fatigue crack initiation," *International journal of fatigue,* vol. 57, pp. 58-72, 2013.

[4] S. Abdullah, N. Al-Asady, A. Ariffin, and M. Rahman, "A review on finite element analysis approaches in durability assessment of automotive components," *Journal of Applied Sciences,* vol. 8, no. 12, pp. 2192-2201, 2008.

[5] L. Susmel and D. Taylor, "A critical distance/plane method to estimate finite life of notched components under variable amplitude uniaxial/multiaxial fatigue loading," *International Journal of Fatigue,* vol. 38, pp. 7-24, 2012.

[6] M. M. Rahman, A. Ariffin, N. Jamaludin, and C. Haron, "Influence of surface treatements on fatigue life of a free piston linear generator engine components using narrow band approach," *SDHM Structural Durability and Health Monitoring,* vol. 2, no. 2, pp. 69-82, 2006.

[7] M. M. Rahman, A. Ariffin, S. Abdullah, and N. Jamaludin, "Finite element based durability assessment of a free piston linear engine component," *Journal of Structural Durability and Health Monitoring,* vol. 3, no. 1, pp. 1-13, 2007.

[8] M. Rahman, K. Kadirgama, M. Noor, M. Rejab, and S. Kesulai, "Fatigue life prediction of lower suspension arm using strain-life approach," *European Journal of Scientific Research,* vol. 30, no. 3, pp. 437-450, 2009.

[9] R. MM, A. Ariffin, M. R. K. Kadirguma, and M. Noor, "Multiaxial fatigue behavior of cylinder head for a free piston linear engine," *Journal of Applied Sciences,* vol. 9, no. 15, pp. 2725-2734, 2009.

[10] M. Kamal and M. M. Rahman, "Fatigue life estimation models: a state of the art," *International Journal of Automotive and Mechanical Engineering,* vol. 9, pp. 1599-1608, 2014.

[11] M. Kamal and M. M. Rahman, "Advances in fatigue life modeling: A review," *Renewable and Sustainable Energy Reviews,* vol. 82, pp. 940-949, 2018.

[12] M. Sharaf, P. Kucharczyk, N. Vajragupta, S. Münstermann, A. Hartmaier, and W. Bleck, "Modeling the microstructure influence on fatigue life variability in structural steels," *Computational Materials Science,* vol. 94, pp. 258-272, 2014/11/01/ 2014.

[13] R. Brighenti and A. Carpinteri, "A notch multiaxial-fatigue approach based on damage mechanics," *International Journal of Fatigue,* vol. 39, pp. 122-133, 2012.

[14] L. Susmel and D. Taylor, "The modified Wöhler curve method applied along with the theory of critical

distances to estimate finite life of notched components subjected to complex multiaxial loading paths," *Fatigue & Fracture of Engineering Materials & Structures,* vol. 31, no. 12, pp. 1047-1064, 2008.

[15] E. Macha and A. Niesłony, "Critical plane fatigue life models of materials and structures under multiaxial stationary random loading: The state-of-the-art in Opole Research Centre CESTI and directions of future activities," *International Journal of Fatigue,* vol. 39, pp. 95-102, 2012.

[16] B. Atzori, F. Berto, P. Lazzarin, and M. Quaresimin, "Multi-axial fatigue behaviour of a severely notched carbon steel," *International Journal of Fatigue,* vol. 28, no. 5-6, pp. 485-493, 2006.

[17] L. Susmel and R. Tovo, "Estimating fatigue damage under variable amplitude multiaxial fatigue loading," *Fatigue & Fracture of Engineering Materials & Structures,* vol. 34, no. 12, pp. 1053-1077, 2011.

[18] M. Rahman, A. Ariffin, S. Abdullah, M. Noor, R. Bakar, and M. Maleque, "Finite element based fatigue life prediction of cylinder head for two-stroke linear engine using stress-life approach," *Journal of Applied Sciences,* vol. 8, no. 19, pp. 3316-3327, 2008.

[19] M. Rahman, A. Ariffin, S. Abdullah, M. Noor, and A. Rosli, "Durability assessment of cylinder block for two stroke free piston linear engine using random loading," *American Journal of Applied Sciences,* vol. 6, no. 4, pp. 726-735, 2009.

[20] A. Scattina *et al.*, "Prediction of Cyclic Fatigue Life of Nickel-Titanium Rotary Files by Virtual Modeling and Finite Elements Analysis," *Journal of endodontics,* vol. 41, no. 11, pp. 1867-1870, 2015.

[21] J. Papuga, "A survey on evaluating the fatigue limit under multiaxial loading," *International Journal of Fatigue,* vol. 33, no. 2, pp. 153-165, 2011.

[22] N. W. Bishop and F. Sherratt, *Finite element based fatigue calculations*. NAFEMS, 2000.

[23] M. Mashiachidi and D. Desai, "Fatigue life prediction of mistuned steam turbine blades subjected to deviations in blade geometry," *Science, Engineering and Technology,* vol. 3, no. 2, pp. 24-35, 2023.

[24] Q. Xin, *Diesel engine system design*. Elsevier, 2011.

[25] D. Deluca, "Understanding fatigue," *ASME International Gas Turbine Institute,* pp. 7-10, 2001.

[26] S. A. Kanth, G. A. Harmain, and A. Jameel, "Investigation of fatigue crack growth in engineering components containing different types of material irregularities by XFEM," *Mechanics of Advanced Materials and Structures,* vol. 29, no. 24, pp. 3570-3587, 2022/08/23 2022.

[27] J. M. E. Marques, D. Benasciutti, A. Niesłony, and J. Slavič, "An overview of fatigue testing systems for metals under uniaxial and multiaxial random loadings," *Metals,* vol. 11, no. 3, p. 447, 2021.

[28] M. Liakat and M. M. Khonsari, "An experimental approach to estimate damage and remaining life of metals under uniaxial fatigue loading," *Materials & Design,* vol. 57, pp. 289-297, 2014.

[29] G. Robak and T. Łagoda, "Variability of fatigue parameters under uniaxial loading in the function of the number of cycles to failure," *International Journal of Fatigue,* vol. 113, pp. 246-252, 2018.

[30] A. Ince and G. Glinka, "A generalized fatigue damage parameter for multiaxial fatigue life prediction under proportional and non-proportional loadings," *International Journal of Fatigue,* vol. 62, pp. 34-41, 2014.

[31] P. Arora, S. K. Gupta, M. K. Samal, and J. Chattopadhyay, "Multiaxial fatigue tests under variable strain paths and asynchronous loading and assessment of fatigue life using critical plane models," *International Journal of Fatigue,* vol. 145, p. 106049, 2021.

[32] A. Fatemi and N. Shamsaei, "Multiaxial fatigue: An overview and some approximation models for life estimation," *International Journal of Fatigue,* vol. 33, no. 8, pp. 948-958, 2011.

[33] M. A. Meggiolaro and J. T. P. de Castro, "An improved multiaxial rainflow algorithm for non-proportional stress or strain histories–Part I: Enclosing surface methods," *International Journal of Fatigue,* vol. 42, pp. 217-226, 2012.

[34] T. Zarrin-Ghalami and A. Fatemi, "Multiaxial fatigue and life prediction of elastomeric components," *International Journal of Fatigue,* vol. 55, pp. 92-101, 2013.

[35] A. Niesłony and C. M. Sonsino, *Comparison of some selected multiaxial fatigue assessment criteria*. Fraunhofer-Inst. für Betriebsfestigkeit und Systemzuverlässigkeit LBF, 2008.

[36] E. Macha, "Generalization of Fatigue Failure Criteria for Multiaxial Sinusoidal Loadings in the Range of Random Loadings," in *ICBMFF2*, 1989.

[37] A. Karolczuk and E. Macha, "A review of critical plane orientations in multiaxial fatigue failure criteria of metallic materials," *International Journal of Fracture,* vol. 134, no. 3-4, p. 267, 2005.

[38] T. Łagoda and P. Ogonowski, "Criteria of multiaxial random fatigue based on stress, strain and energy parameters of damage in the critical plane," *Materialwissenschaft und Werkstofftechnik: Entwicklung, Fertigung, Prüfung, Eigenschaften und Anwendungen technischer Werkstoffe,* vol. 36, no. 9, pp. 429-437, 2005.

[39] D. Socie and G. Marquis, "Multiaxial Fatigue, Society of Automotive Engineers," *Inc. Warrendale, Pa,* vol. 484, 2000.

[40] A. Niesłony, "Determination of fragments of multiaxial service loading strongly influencing the fatigue of machine components," *Mechanical Systems and Signal Processing,* vol. 23, no. 8, pp. 2712-2721, 2009.

[41] E. Macha, "Fatigue failure criteria for materials under random triaxial state of stress," in *Fracture 84*: Elsevier, 1984, pp. 1895-1902.

[42] Z. Qi, N. Hu, Z. Li, D. Zeng, and X. Su, "A stress-based model for fatigue life prediction of high density polyethylene under complicated loading conditions,"

International Journal of Fatigue, vol. 119, pp. 281-289, 2019/02/01/ 2019.
[43] Z. Engin and D. Coker, "Comparison of Equivalent Stress Methods with Critical Plane Approaches for Multiaxial High Cycle Fatigue Assessment," *Procedia Structural Integrity,* vol. 5, pp. 1229-1236, 2017/01/01/ 2017.
[44] G. Sines, "Failure of materials under combined repeated stresses with superimposed static stresses," California. Univ., Los Angeles1955.
[45] G. Sines, "Behavior of metals under complex static and alternating stresses," *Metal fatigue,* vol. 1, pp. 145-169, 1959.
[46] H. S. Googarchin, S. Sharifi, F. Forouzesh, G. Hosseinpour, S. Etesami, and S. M. Zade, "Comparative study on the fatigue criteria for the prediction of failure in engine structure," *Engineering Failure Analysis,* vol. 79, pp. 714-725, 2017.
[47] B. Crossland, "Effect of large hydrostatic pressures on the torsional fatigue strength of an alloy steel," in *Proc. Int. Conf. on Fatigue of Metals*, 1956, vol. 138: Institution of Mechanical Engineers London.
[48] W. N. Findley, *A theory for the effect of mean stress on fatigue of metals under combined torsion and axial load or bending* (no. 6). Engineering Materials Research Laboratory, Division of Engineering, Brown University, 1958.
[49] D. McDiarmid, "A Shear Stress Based Critical-plane Criterion of Multiaxial Fatigue Failure for Design and Life Prediction," *Fatigue & Fracture of Engineering Materials & Structures,* vol. 17, no. 12, pp. 1475-1484, 1994.
[50] D. McDiarmid, "A general criterion for high cycle multiaxial fatigue failure," *Fatigue & Fracture of Engineering Materials & Structures,* vol. 14, no. 4, pp. 429-453, 1991.
[51] K. Dang-Van, "Macro-micro approach in high-cycle multiaxial fatigue," in *Advances in multiaxial fatigue*: ASTM International, 1993.
[52] A. A. A. Rahim, C. H. Chin, S. Abdullah, S. S. K. Singh, and N. M. Nor, "Fatigue Life Analysis of Strain Loadings Using Effective Strain Damage Approach with the Consideration of Cycle Sequence Effects," pp. 87-95: Springer.
[53] J. Draper, *Modern metal fatigue analysis*. EMAS publishing, 2008.
[54] F. A. Kandil, M. W. Brown, and K. Miller, "Biaxial low-cycle fatigue failure of 316 stainless steel at elevated temperatures," in *Mechanical behaviour and nuclear applications of stainless steel at elevated temperatures*, 1982.
[55] M. Brown and K. Miller, "Two decades of progress in the assessment of multiaxial low-cycle fatigue life," in *Low-cycle fatigue and life prediction*: ASTM International, 1982.
[56] M. W. Brown and K. Miller, "A theory for fatigue failure under multiaxial stress-strain conditions," *Proceedings of the Institution of Mechanical Engineers,* vol. 187, no. 1, pp. 745-755, 1973.
[57] C. Wang and M. Brown, "A path-independent parameter for fatigue under proportional and non-proportional loading," *Fatigue & fracture of engineering materials & structures,* vol. 16, no. 12, pp. 1285-1297, 1993.
[58] K. N. Smith, "A stress-strain function for the fatigue of metals," *Journal of materials,* vol. 5, pp. 767-778, 1970.
[59] Z.-R. Wu, X.-T. Hu, and Y.-D. Song, "Multiaxial fatigue life prediction for titanium alloy TC4 under proportional and nonproportional loading," *International Journal of Fatigue,* vol. 59, pp. 170-175, 2014.
[60] L. Wang, S. Shi, S. Fu, G. Chen, and X. Chen, "Evaluation of multiaxial fatigue life prediction criteria for PEEK," *Theoretical and Applied Fracture Mechanics,* vol. 73, pp. 128-135, 2014.
[61] J. Albinmousa and H. Jahed, "Multiaxial effects on LCF behaviour and fatigue failure of AZ31B magnesium extrusion," *International Journal of Fatigue,* vol. 67, pp. 103-116, 2014.
[62] S. Babaei, A. Ghasemi-Ghalebahman, and R. Hajighorbani, "A fatigue model for sensitive materials to non-proportional loadings," *International journal of fatigue,* vol. 80, pp. 266-277, 2015.
[63] A. Fatemi and D. F. Socie, "A critical plane approach to multiaxial fatigue damage including out-of-phase loading," *Fatigue & Fracture of Engineering Materials & Structures,* vol. 11, no. 3, pp. 149-165, 1988.
[64] R. I. Stephens, A. Fatemi, R. R. Stephens, and H. O. Fuchs, *Metal fatigue in engineering*. John Wiley & Sons, 2000.
[65] M. Gladskyi and A. Fatemi, "Notched fatigue behavior including load sequence effects under axial and torsional loadings," *International journal of fatigue,* vol. 55, pp. 43-53, 2013.
[66] K. Liu and J. Wang, "An energy method for predicting fatigue life, crack orientation, and crack growth under multiaxial loading conditions," *International Journal of Fatigue,* vol. 23, pp. 129-134, 2001.
[67] S. Calvo, M. Canales, C. Gómez, J. Valdés, and J. Núñez, "Probabilistic formulation of the multiaxial fatigue damage of Liu," *International Journal of Fatigue,* vol. 33, no. 3, pp. 460-465, 2011.
[68] C.-C. Chu, "Fatigue damage calculation using the critical plane approach," *Journal of Engineering Materials and Technology,* vol. 117, no. 1, pp. 41-49, 1995.
[69] G. Glinka, G. Wang, and A. Plumtree, "Mean stress effects in multiaxial fatigue," *Fatigue & fracture of engineering materials & structures,* vol. 18, no. 7-8, pp. 755-764, 1995.
[70] Z.-Y. Yu, S.-P. Zhu, Q. Liu, and Y. Liu, "A new energy-critical plane damage parameter for multiaxial

fatigue life prediction of turbine blades," *Materials,* vol. 10, no. 5, p. 513, 2017.
[71] V. Bonnand, J. Chaboche, H. Cherouali, P. Kanoute, E. Ostoja-Kuczynski, and F. Vogel, "Investigation of multiaxial fatigue in the prospect of turbine disc applications: Part II–Fatigue criteria analysis and formulation of a new combined one," in *ICMFF9*, 2010.
[72] N. Shamsaei and A. Fatemi, "Effect of hardness on multiaxial fatigue behaviour and some simple approximations for steels," *Fatigue & Fracture of Engineering Materials & Structures,* vol. 32, no. 8, pp. 631-646, 2009.
[73] A. Banvillet, T. Łagoda, E. Macha, A. Niesłony, T. Palin-Luc, and J.-F. Vittori, "Fatigue life under non-Gaussian random loading from various models," *International Journal of Fatigue,* vol. 26, no. 4, pp. 349-363, 2004.
[74] H. Wei and Y. Liu, "A critical plane-energy model for multiaxial fatigue life prediction," *Fatigue & Fracture of Engineering Materials & Structures,* vol. 40, no. 12, pp. 1973-1983, 2017.
[75] B.-R. You and S.-B. Lee, "A critical review on multiaxial fatigue assessments of metals," *International Journal of Fatigue,* vol. 18, no. 4, pp. 235-244, 1996.
[76] H. Gough, "Engineering steels under combined cyclic and static stresses," *Journal of Applied Mechanics-Transactions of the ASME,* vol. 17, no. 2, pp. 113-125, 1950.
[77] P. Lazzarin and L. Susmel, "A stress-based method to predict lifetime under multiaxial fatigue loadings," *Fatigue & Fracture of Engineering Materials & Structures,* vol. 26, no. 12, pp. 1171-1187, 2003.
[78] L. Susmel and D. Taylor, "A simplified approach to apply the theory of critical distances to notched components under torsional fatigue loading," *International Journal of Fatigue,* vol. 28, no. 4, pp. 417-430, 2006.
[79] L. Susmel and D. Taylor, "The theory of critical distances to estimate lifetime of notched components subjected to variable amplitude uniaxial fatigue loading," *International Journal of Fatigue,* vol. 33, no. 7, pp. 900-911, 2011.
[80] L. Susmel, R. Tovo, and D. F. Socie, "Estimating the orientation of Stage I crack paths through the direction of maximum variance of the resolved shear stress," *International Journal of Fatigue,* vol. 58, pp. 94-101, 2014.
[81] R. Louks, B. Gerin, J. Draper, H. Askes, and L. Susmel, "On the multiaxial fatigue assessment of complex three-dimensional stress concentrators," *International Journal of Fatigue,* vol. 63, pp. 12-24, 2014.
[82] Y. Liu and S. Mahadevan, "Multiaxial high-cycle fatigue criterion and life prediction for metals," *International Journal of Fatigue,* vol. 27, no. 7, pp. 790-800, 2005.
[83] Y. Liu and S. Mahadevan, "A unified multiaxial fatigue damage model for isotropic and anisotropic materials," *International Journal of Fatigue,* vol. 29, no. 2, pp. 347-359, 2007.
[84] D. Ninic and H. L. Stark, "A multiaxial fatigue damage function," *International Journal of Fatigue,* vol. 29, no. 3, pp. 533-548, 2007.
[85] J. Papuga and M. Růžička, "Two new multiaxial criteria for high cycle fatigue computation," *International Journal of Fatigue,* vol. 30, no. 1, pp. 58-66, 2008.
[86] Z. Lu and Y. Liu, "Crack growth-based multiaxial fatigue life prediction," in *ICF12, Ottawa 2009*, 2009.
[87] G.-Q. Sun, D.-G. Shang, and M. Bao, "Multiaxial fatigue damage parameter and life prediction under low cycle loading for GH4169 alloy and other structural materials," *International Journal of Fatigue,* vol. 32, no. 7, pp. 1108-1115, 2010.
[88] J. Li, Q. Sun, Z.-P. Zhang, C.-W. Li, and D.-W. Zhang, "A new multiaxial fatigue life prediction model under proportional and nonproportional loading," *Journal of Engineering Materials and Technology,* vol. 132, no. 2, p. 021016, 2010.
[89] J. Li, Z.-p. Zhang, Q. Sun, and C.-w. Li, "Multiaxial fatigue life prediction for various metallic materials based on the critical plane approach," *International Journal of fatigue,* vol. 33, no. 2, pp. 90-101, 2011.
[90] C. Ronchei, A. Carpinteri, G. Fortese, A. Spagnoli, S. Vantadori, and M. Kurek, "Life estimation by varying the critical plane orientation in the modified Carpinteri-Spagnoli criterion," *Frattura Ed Integrita Strutturale,* no. 34, 2015.
[91] V. Anes, L. Reis, B. Li, M. Fonte, and M. De Freitas, "New approach for analysis of complex multiaxial loading paths," *International Journal of Fatigue,* vol. 62, pp. 21-33, 2014.
[92] V. Anes, L. Reis, B. Li, M. Freitas, and C. Sonsino, "Minimum Circumscribed Ellipse (MCE) and Stress Scale Factor (SSF) criteria for multiaxial fatigue life assessment," *Theoretical and Applied Fracture Mechanics,* vol. 73, pp. 109-119, 2014.
[93] V. Anes, L. Reis, B. Li, and M. De Freitas, "New cycle counting method for multiaxial fatigue," *International journal of fatigue,* vol. 67, pp. 78-94, 2014.
[94] P. Lopez-Crespo, B. Moreno, A. Lopez-Moreno, and J. Zapatero, "Study of crack orientation and fatigue life prediction in biaxial fatigue with critical plane models," *Engineering Fracture Mechanics,* vol. 136, pp. 115-130, 2015.
[95] M. Mohammadi, M. Zehsaz, S. Hassanifard, and A. Rahmatfam, "An evaluation of total fatigue life prediction of a notched shaft subjected to cyclic bending load," *Engineering Fracture Mechanics,* vol. 166, pp. 128-138, 2016.
[96] A. Carpinteri, A. Spagnoli, C. Ronchei, D. Scorza, and S. Vantadori, "Critical plane criterion for fatigue life calculation: time and frequency domain formulations," *Procedia Engineering,* vol. 101, pp. 518-523, 2015.
[97] L. Liu, Q. Guo, and T. He, "Thermal-acoustic fatigue of a multilayer thermal protection system in combined

extreme environments," *Advances in Mechanical Engineering,* vol. 6, p. 176891, 2014.

[98] J. Ge, Y. Sun, S. Zhou, L. Zhang, Y. Zhang, and Q. Zhang, "A hybrid frequency–time domain method for predicting multiaxial fatigue life of 7075-T6 aluminium alloy under random loading," *Fatigue & Fracture of Engineering Materials & Structures,* vol. 38, no. 3, pp. 247-256, 2015.

[99] J. Ge, Y. Sun, J. Xu, Z. Yang, and J. Liang, "Fatigue life prediction of metal structures subjected to combined thermal-acoustic loadings using a new critical plane model," *International Journal of Fatigue,* vol. 96, pp. 89-101, 2017.

[100] C. A. Gonçalves, J. A. Araújo, and E. N. Mamiya, "Multiaxial fatigue: a stress based criterion for hard metals," *International Journal of Fatigue,* vol. 27, no. 2, pp. 177-187, 2005.

[101] L. Khalij, E. Pagnacco, and S. Lambert, "A measure of the equivalent shear stress amplitude from a prismatic hull in the principal coordinate system," *International Journal of Fatigue,* vol. 32, no. 12, pp. 1977-1984, 2010.

[102] J. Araújo, A. Dantas, F. Castro, E. Mamiya, and J. Ferreira, "On the characterization of the critical plane with a simple and fast alternative measure of the shear stress amplitude in multiaxial fatigue," *International Journal of Fatigue,* vol. 33, no. 8, pp. 1092-1100, 2011.

[103] E. Mamiya, F. Castro, R. Algarte, and J. Araújo, "Multiaxial fatigue life estimation based on a piecewise ruled S–N surface," *International Journal of Fatigue,* vol. 33, no. 4, pp. 529-540, 2011.

[104] M. A. Meggiolaro and J. T. P. de Castro, "An improved multiaxial rainflow algorithm for non-proportional stress or strain histories–Part II: The Modified Wang–Brown method," *International Journal of Fatigue,* vol. 42, pp. 194-206, 2012.

[105] G. Marsh *et al.*, "Review and application of Rainflow residue processing techniques for accurate fatigue damage estimation," *International Journal of Fatigue,* vol. 82, pp. 757-765, 2016.

[106] I. V. Papadopoulos, "A new criterion of fatigue strength for out-of-phase bending and torsion of hard metals," *International Journal of Fatigue,* vol. 16, no. 6, pp. 377-384, 1994.

[107] I. V. Papadopoulos, P. Davoli, C. Gorla, M. Filippini, and A. Bernasconi, "A comparative study of multiaxial high-cycle fatigue criteria for metals," *International Journal of Fatigue,* vol. 19, no. 3, pp. 219-235, 1997.

[108] T. Palin-Luc and S. Lasserre, "An energy based criterion for high cycle multiaxial fatigue," *European Journal of Mechanics-A/Solids,* vol. 17, no. 2, pp. 237-251, 1998.

[109] A. Banvillet, T. Palin-Luc, and S. Lasserre, "A volumetric energy based high cycle multiaxial fatigue citerion," *International Journal of Fatigue,* vol. 25, no. 8, pp. 755-769, 2003.

[110] N. Saintier, T. Palin-Luc, J. Bénabes, and F. Cocheteux, "Non-local energy based fatigue life calculation method under multiaxial variable amplitude loadings," *International Journal of Fatigue,* vol. 54, pp. 68-83, 2013.

[111] H. Zenner, A. Simbürger, and J. Liu, "On the fatigue limit of ductile metals under complex multiaxial loading," *International Journal of Fatigue,* vol. 22, no. 2, pp. 137-145, 2000.

[112] C. Luo and A. Chattopadhyay, "Prediction of fatigue crack initial stage based on a multiscale damage criterion," *International Journal of Fatigue,* vol. 33, no. 3, pp. 403-413, 2011.

[113] M. F. Horstemeyer and A. M. Gokhale, "A void–crack nucleation model for ductile metals," *International Journal of Solids and Structures,* vol. 36, no. 33, pp. 5029-5055, 1999.

[114] M. Lugo, J. Jordon, M. Horstemeyer, M. Tschopp, J. Harris, and A. Gokhale, "Quantification of damage evolution in a 7075 aluminum alloy using an acoustic emission technique," *Materials Science and Engineering: A,* vol. 528, no. 22-23, pp. 6708-6714, 2011.

[115] Q. Vu, D. Halm, and Y. Nadot, "Multiaxial fatigue criterion for complex loading based on stress invariants," *International Journal of Fatigue,* vol. 32, no. 7, pp. 1004-1014, 2010.

[116] H. Pinto, A. M. De Jesus, A. Fernández-Canteli, E. Castillo, and H. F. Pereira, "Analysis of constant and variable amplitude strain-life data using a novel probabilistic Weibull regression model," *Journal of Pressure Vessel Technology,* vol. 132, no. 6, p. 061401, 2010.

[117] O. Scott-Emuakpor, T. George, C. Cross, J. Wertz, and M.-H. H. Shen, "A new distortion energy-based equivalent stress for multiaxial fatigue life prediction," *International Journal of Non-Linear Mechanics,* vol. 47, no. 3, pp. 29-37, 2012.

[118] M. Chaussumier, C. Mabru, M. Shahzad, R. Chieragatti, and F. Rezai-Aria, "A predictive fatigue life model for anodized 7050 aluminium alloy," *International Journal of Fatigue,* vol. 48, pp. 205-213, 2013.

[119] K. Khandelwal and S. El-Tawil, "A finite strain continuum damage model for simulating ductile fracture in steels," *Engineering Fracture Mechanics,* vol. 116, pp. 172-189, 2014.

[120] S. T. Bukkapatnam and K. Sadananda, "A genetic algorithm for unified approach-based predictive modeling of fatigue crack growth," *International Journal of Fatigue,* vol. 27, no. 10-12, pp. 1354-1359, 2005.

[121] Y. Liu, "Stochastic modeling of multiaxial fatigue and fracture," 2006.

[122] A. P. Vassilopoulos, E. F. Georgopoulos, and T. Keller, "Comparison of genetic programming with conventional methods for fatigue life modeling of FRP composite materials," *International Journal of Fatigue,* vol. 30, no. 9, pp. 1634-1645, 2008.

[123] M. Franulović, R. Basan, and I. Prebil, "Genetic algorithm in material model parameters' identification

for low-cycle fatigue," *Computational Materials Science,* vol. 45, no. 2, pp. 505-510, 2009.

[124] K. Krishnapillai and R. Jones, "Three-dimensional structural design optimisation based on fatigue implementing a genetic algorithm and a non-similitude crack growth law," *Finite Elements in Analysis and Design,* vol. 45, no. 2, pp. 132-146, 2009.

[125] M. Kamal, M. M. Rahman, and M. Sani, "Fatigue life prediction using simplified endurance function model," *Advances in Mechanical Engineering,* vol. 5, p. 581754, 2013.

[126] C. Roux, X. Lorang, H. Maitournam, M.-L. Nguyen-Tajan, and B. Quesson, "Multi-parameter Fatigue Equivalence Loadings for specification applications," *Procedia Engineering,* vol. 66, pp. 393-402, 2013.

[127] B. Lotfi and P. Beiss, "Application of neural networking for fatigue limit prediction of powder metallurgy steel parts," *Materials & Design,* vol. 50, pp. 440-445, 2013.

[128] A. Niesłony and M. Böhm, "Mean stress effect correction using constant stress ratio S–N curves," *International journal of fatigue,* vol. 52, pp. 49-56, 2013.

[129] M. Dorigo, V. Maniezzo, and A. Colorni, "Ant system: optimization by a colony of cooperating agents," *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics),* vol. 26, no. 1, pp. 29-41, 1996.

[130] P. Korošec, J. Šilc, and B. Filipič, "The differential ant-stigmergy algorithm," *Information Sciences,* vol. 192, pp. 82-97, 2012.

[131] J. Klemenc and M. Fajdiga, "Estimating S–N curves and their scatter using a differential ant-stigmergy algorithm," *International journal of fatigue,* vol. 43, pp. 90-97, 2012.

[132] J. Klemenc and M. Fajdiga, "Joint estimation of E–N curves and their scatter using evolutionary algorithms," *International Journal of Fatigue,* vol. 56, pp. 42-53, 2013.

Appendix

Table A1. List of symbols for the different models and their description.

Model	Symbols	Description	Model	Symbols	Description
Fatigue analysis models utilizing strain energy-based approach	G	Shear modulus	Assessment of multiaxial fatigue using the critical plane model	S, A and B	Material constants
	E	Elastic modulus		c	Fatigue ductility exponent
	c	Coefficient of fatigue ductility exponent		b	Fatigue strength coefficient
	b	Coefficient of fatigue strength		$\Delta\gamma_{max}$	Maximum shear strain range
	$\Delta\tau$	Shear stress range range		$\Delta\acute{\gamma}$	Equivalent shear strain connection
	$\Delta\gamma$	Shear strain range		$\sigma_{n,\ mean}$	Mean stress
	ΔW	Virtual strain energy parameter		ε_n^*	Normal extrusion strain between two turning points of γ_{max}
	$\Delta\varepsilon_1$	Principal strain range		v_p	Plastic Poisson ratio
	$\tau_{n,max}$	Maximum shear stress (on the plate)		v_e	Elastic Poisson ratio
	τ_f'	Shear fatigue strength		MDP_{WB}	Wang-Brown Maximum damage parameter
	σ_y	Shear fatigue strength coefficient		$\Delta\varepsilon_n$	Nominal strain range
	$\sigma_{n,max}$	Maximum nominal stress		$\Delta\gamma_{max}$	Maximum shear range
	σ_f'	Fatigue strength coefficient		t	Torsional fatigue
	ε_f'	Coefficient of fatigue ductility		s	Mode II and Mode I intensity factor ratio
	v_p	Plastic Poisson ratio		n	Empirical constant between points 1 and 2
	N_f	Fatigue life		N	Fatigue life
	c_γ	Coefficient of shear fatigue ductility exponent		m	Mean stress sensitivity index
	b_γ	Coefficient of shear fatigue exponent		l, m, n	Direction cosine of a vector normal to the plane
	$\Delta\sigma_n$	Normal stress range		L	Material characteristics length
	$\Delta\varepsilon_n$	Normal strain range		D	Damage on the critical plane
Fatigue analysis models using equivalent stress	$\tau(t)$	Instantaneous shear stress		d	Direction cosines
	a, b, k, f, α and β	Material constants		c	Fatigue ductility exponent
	$\Delta\tau$	Shear stress range		b_f	Bending fatigue
	σ_{uts}	Ultimate tensile strength		b	Fatigue strength coefficient
	σ_n	Nominal stress		$a, b, m\ A, B, C, k, a_c, b_c, \alpha$ and β, μ	Material constants
	$\sigma_h(t)$	Instantaneous hydrostatic stress		a	Crack length
	σ_h	Hydrostatic stress		$\Delta\tau$	Shear stress range
	$t_{A,B}$	Shear stress fatigue strength		$\emptyset$	Phase angle
	$\Delta\tau_{oct}$	Octahedral von-Misis stress range		τ_{max}	Maximum shear stress
	$\Delta\tau_{max}$	Maximum shear stress range		τ_a	Shear amplitude
Fatigue analysis models using equivalent strain	S, A and B	Material constants		τ_a	Torsional stress
	c	Fatigue ductility exponent		$\tau_{a,c}$	Shear stress amplitude on the critical plane
	b	Fatigue strength coefficient		$\tau_{A,\ Ref}(\rho_{eff})$	Reference shear stress amplitude (at a defined limit to failure cycle)
	$\Delta\gamma_{max}$	Maximum shear strain range		σ_u	Material's tensile strength

	$\Delta\acute{\gamma}$	Equivalent shear strain connection		$\sigma_{n,a}$	Stress amplitude (perpendicular to the critical plane)
	$\sigma_{n,\ mean}$	Mean stress		$\sigma_{n,max}$	Maximum normal stress
	ε_n^*	Normal extrusion strain between two turning points of γ_{max}		$\sigma_{n,m}$	Mean stress perpendicular to the critical plane
	v_p	Plastic Poisson ratio		σ_m	Bending mean stress
	v_e	Elastic Poisson ratio		$\sigma_{m,c}$	Mean normal stress (on the critical plane)
	MDP_{WB}	Wang-Brown Maximum damage parameter		σ_{eq}	Equivalent stress amplitude
	$\Delta\varepsilon_n$	Nominal strain range		σ_a	Bending stress
	$\Delta\gamma_{max}$	Maximum shear range		$\sigma_{a,c}^H$	Hydrostatic stress amplitude (on the critical plane)
Fatigue analysis based on enclosed surface model	φ	Angle locating rectangular hull on plane		$\sigma_{a,c}$	Normal stress amplitude (on the critical plane)
	δ	Material parameter		ρ_{eff}	Critical plane stress
	α, β,γ,δ	Material parameters		ε_n^*	Normal strain exercusion (occurring between turning points on the critical plane)
	k	Fatigue limit ratio		W_{gen}^*	Maximum generalized strain energy
	τ_a^{max}	Maximum equivalent shear stress amplitude		v_{eff}	Effective poisson ratio
	$\tau_a(\emptyset,\theta)$	Equivalent shear stress on plane located by $\emptyset$ and θ		T_e	Fully reversed torsion fatigue limit
	$\sigma_{p\ max}$	Maximum principal stress		t_{N_f}	Shear stress at N_f cycles
	$R_1 - R_5$	Stress amplitude in 5D Euclidean space in the principal directions		S_e	Fully reversed axial and torsion fatigue limit
	N_f	Fatigue life		N_a	Maximum normal stress
	$a_{(1-5)}$	Deviatoric stress amplitude		$k_\tau(\rho_{eff})$	Wholer's curve negative inverse slope
	$\sqrt{J_{2,a}}$	Second invariant stress deviator amplitude		$K_{mixed,eq}$	Equivalent stress intensity factor under general mixed mode loading
Fatigue assessment using material structure models	$\sigma_s^{(\alpha')}$	Shear stress (obtained on the slip system (α))		k_1, k_2 and k^H	Loading related parameters
	σ_o	Endurance limit		k'	Normal stress sensitivity factor
	$\sigma_n^{(\alpha)}$	Normal stress (obtained on the slip system (α))		f_{-1}	Fully reversed axial loading fatigue limit
	σ_{mr}	Memory stress		f_{N_f}	Normal stress at N_f cycles
	σ_f	True fracture stress		d_1 and d_2	Fatigue material properties to be experimentally determined
	$m'$and δ'	Material constants		c_a	Shear stress on the considered plane
	$d\varepsilon^p$	Plastic strain increment corresponding to $\sigma_n^{(\alpha')}$		$[C]$	Matrix: consisting of (i) variance V_i and (ii) covariance $C_{i,j}$
	$d\gamma^p$	Plastic strain increment corresponding to $\sigma_s^{(\alpha')}$		$\Delta\sigma_o$	Uniaxial plane fatigue limits range
	$dY^{(\alpha')}$	Plastic strain energy increment (obtained on slip system(α))		$\Delta\sigma_n$	Normal stress range
	$dD^{(\alpha)}$	Damage parameter increment on slip system(α)		$\Delta\varepsilon_n^p$	Plastic normal strain range
Stress invariant-based fatigue assessment model	$\epsilon(t)$	Nucleation of void nucleation as a function of time, t		$\Delta\varepsilon_n^e$	Elastic normal strain range
	$\varepsilon(t)$	Strain as a function of time, t		$\Delta\varepsilon_{gen}^*$	Maximum generalized strain amplitude

	f	Equivalent stress amplitude
	d	Length scale parameter
	t_{-1}	Fully reversed torsion Fatigue limit
	K_{IC}	Critical stress intensity factor
	J_2'	Stress tensor amplitude deviator in the secondary invariant
	J_2	Devitoric stress invariants
	$J_{2,mean}$	Mean value of $J_2'(t)$ as a function of time
	I_f	Function of $I_{1,a}$ and $I_{1,m}$
	I_1	Stress invariant
	I_1	First stress invariant (consisting of m – mean and a – amplitude)
	f_{-1}	Fully reversed axial and bending fatigue limit
	a', b' and c', $\gamma_1, \gamma_2, \gamma_3$, C_{coeff} and β	Material constants
Plasticity-based models for fatigue assessment	σP	Principal stress (at P = 1, 2, 3)
	T	Stress tri-axiality
	$\Delta\varepsilon$	Smoothing factor
	σ_E	Non-linear equivalent stress
	σ_C	Cyclic stress material parameter
	ε_n	Nucleation strain
	ε_c	Coalescence strain
	α^P	Plastic internal variable
	T_1 and T_2	Threshold function
	a_0, a_1, a_2, a_3	Material parameters
	$\dot{\alpha}^d$	Damage variable

	$\Delta\gamma^p$	Plastic strain range
	$\Delta\gamma_{max}$	Maximum shear strain range
	$\Delta\gamma^e$	Elastic strain range
	ΔK_{th}	Threshold stress factor
	$\Delta K_{I,th}$	Fatigue threshold intensity factor
	ΔK_{eq}	Equivalent stress intensity factor
Fatigue analysis based on integral type models	φ and θ	Angle locating the plane
	ϑ and φ	Euler angles of planes (examined in the local coordinate system)
	x	The angle between resolved the shear stress and the major axis
	$a, m, b, n, k, a_1, b_1, \alpha$ and β	Material constants
	ω_a^D	Strain energy density volumetric mean value
	$\tau_{\gamma\varphi\alpha}$	Alternating shear stress on plane $\gamma\varphi$
	$\tau_{\gamma\varphi m}$	Static shear stress on plane $\gamma\varphi$
	$\sigma_{\gamma\varphi\alpha}$	Alternating normal stress on plane $\gamma\varphi$
	$\sigma_{\gamma\varphi m}$	Normal stress on plane $\gamma\varphi$
	$\sigma_{equ,\alpha}$	Equivalent stress amplitude
	α_M	Stress/strain Sequence duration and stress//strain tensors evolution
	W_{geqdam}	Strain work density damaging part
	W_{geq}	Equivalent strain work density
	W_g^*	Minimum strain work volumetric density required to produce irreversible damage
	W_a^*	Strain energy volumetric density (at a critical point C_i)
	W_a	Strain volumetric energy density
	$V^*(C_i)$	Volumetric critical point
	T_a	Amplitude of resolved shear stress
	t_{-1}	Fully reversed torsion ftigue limit
	P_l	Proportionality factor
	N_m	Mean normal stress
	N_a	Maximum normal stress
	f_{-1}	Fully reversed axial loading fatigue limit
	C_a	Shear stress on the considered plane