Performances improvement of DC Motor using a Fractional Order Adaptive PID Controller optimized by Genetic Algorithm

Abdelouaheb Boukhalfa[1,2], Yassine Bensafia[3], Khatir Khettab[1,4]

[1]*Electrical Engineering Department, University of M'sila, University Pole, Road Bourdj Bou Arreiridj, M'sila 28000 Algeria.*
[2]*QUERE laboratory, Sétif-1University, 19000 – Algeria.*
[3]*LISEA Laboratory, Department of Electrical Engineering, Sciences and Applied Sciences Faculty, Bouira University, Algeria.*
[4]*GE laboratory, M'sila University, University Pole, Road Bourdj Bou Arreiridj, M'sila 28000 Algeria.*

Abstract

In the past 20 years, scientists and engineers have rediscovered fractional calculus and have begun using it in more and more domains, most notably control theory. This study introduces a fractional adaptive PID (FAPID) controller which incorporates an additional parameter to enhance the performance of a conventional adaptive PID (APID) controller. A comparative analysis is conducted between the APID and FAPID controllers optimized using the metaheuristic Genetic Algorithm (GA). The evaluation uses a linearized model of the DC motor control system. The results demonstrate that FAPID controllers significantly outperform conventional APID controllers, particularly regarding rise time, settling time, overshoot, and mean absolute error. Among the proposed designs, the integration of FAPID proves to be the most effective in achieving a balance between responsiveness and stability, exhibiting exceptional robustness and adaptability to variations in DC motor and environmental conditions. This method can be extended to various fractional and integer systems to enhance their efficiency and reduce noise disturbance.

Keywords: *Integer Adaptive PID, Genetic Algorithm, DC motor, Fractional Adaptive PID controllers, Optimization Methods.*

1. Introduction

Following an initial exchange between the Hospital and Leibniz regarding a fractional-order (FO) derivative and its potential implications, fractional calculus emerged as a recognized field of scholarly investigation [1]. Applications of fractional order differentiation have piqued the curiosity of researchers from a wide range of scientific fields, especially the applied sciences. [2], [3], [4]. Podlubny's 1997 development of Fractional-Order Proportional-Integral-Derivative (FOPID) controllers revolutionized control systems by allowing for more flexible and precise tuning [5]. These fractional calculus-based controllers provide greater stability, robustness, and responsiveness, making them ideal for high-precision applications [6].

It is commonly known that one of the best control strategies now in use is the adaptive control method, the main trends in Fractional Order Adaptive PID Control lie in improving the flexibility, robustness, and performance of control systems in the face of complex dynamics and uncertainties. Advancements in adaptive tuning, computational methods, hybridization with AI, and theoretical developments are key driving forces in expanding the applicability of FOAPID controllers across various industries.

Which deals with linear and non-linear systems' parametric uncertainty. Nonetheless, the emphasis was

Corresponding author: Khatir Khettab *(khatir.khettab@univ-msila.dz)*
Received: *19 August 2024*; Revised: *22 December 2024*; Accepted: *27 December 2024*; Published: 8 *January 2025*

on implementing the adaptive control approach using integer order systems [7]. Fractional calculus, a method that generalizes integer-order derivatives and integrals to non-integer orders, has been widely used in systems and control modeling, enhancing controller performance and robustness in fields like robotics [8], process control [9], aircraft systems [10] and biomedical engineering [11].

One clear benefit of fractional-order systems is their use in feedback control; in this case, fractional-order filters can raise system performance. The hereditary nature of fractional-order operators [12] primarily contributes to this improvement, as it enhances the system's resilience against noise and external disturbances [13]. This capacity is particularly beneficial in settings where stability and accuracy are crucial.

PID controllers, essential instruments in control systems, are extensively used to control and vary industrial operations. In order to provide precise and efficient control over dynamic systems, PID controllers combine three fundamental actions: proportional, integral, and derivative, thereby enhancing the stability and responsiveness of the process. Over the past few decades, the fractional-order PID (FOPID) controller has seen rapid development and broad application in control engineering and related sciences [14].

Adaptive control, a leading method to manage parametric uncertainty in both linear and nonlinear systems, has traditionally focused on integer-order systems [15]. However, fractional-order approaches introduce new possibilities, particularly in optimizing system behavior under varying conditions. Genetic algorithms are a potent tool known for their ability to discover optimal or nearly perfect solutions in intricate optimization challenges. They are now commonly used to fine-tune control parameters in fractional systems.

Traditionally, the APID controller has been the industry standard for control applications due to its straightforward design, simplicity, and ease of parameter tuning. While APID controllers offer a robust solution for basic control requirements, they often struggle with complex systems that require high precision and dynamic adaptability. To address these limitations, many modified versions of the APID controller have been introduced over time, yet challenges remain in areas like noise rejection and response flexibility.

A genetic algorithm is a powerful computer tool that may quickly identify exact or approximate answers to complicated search and optimization issues. The process consists of numerous essential processes, including decoding, crossing, evaluation, encoding, and mutation. The procedure starts with a randomly created starting population, and each individual's fitness is then evaluated. The fitness function is critical in the genetic algorithm since it directly affects the algorithm's capacity to attain the desired results. By applying the fitness function to each individual, the algorithm assigns a fitness value that directs population selection and evolution toward optimal solutions [16].

The main highlight of this paper lies in the incorporation of the FAPID controller, refined through genetic algorithm optimization. Its main goal is to improve the performance of DC motor systems by lowering their rise time, settling time, and overshooting. We can get more accurate control by adding fractional-order dynamics to the standard APID structure. The parameters are then optimized using a genetic algorithm that is tailored to the needs of the system. This integrated framework not only allows a thorough study but also lays the way for future advances in the field of fractional control systems.

The rest of the paper is arranged as follows: Section 2 presents a basic understanding of fractional-order systems. Section 3 focuses on the modeling of the DC motor being studied. Section 4 looks at the algorithms used for both integer-order and fractional-order adaptive PID controllers. Section 5 gives a comparative performance study of various controllers using simulations of the DC motor system. Section 6 concludes by reviewing the findings and discussing potential future research options.

2. Fractional Order Systems

2.1. Fractional calculus

Fractional calculus, a branch of calculus theory, extends derivative and integral operations to non-integer orders, expanding calculus's fundamental conceptions. It's a great way to model and test complicated systems because it allows fractional orders. It's also a more accurate way to describe things that happen in the real world than traditional integer-order methods [17]. This unique feature has led to a significant increase in its applications across various sectors, demonstrating its effectiveness in addressing challenges where traditional techniques fall short [18].

More fields can use fractional-order systems now that approximation methods like rational functions for fractional derivatives and integrals are used. These include control theory [19], economics [20], renewable energy [21], etc.

A generalized base operator is used for integration/differentiation, as follows:

$$aD_t^{\alpha} = \begin{cases} \frac{d^{\alpha}}{dt^{\alpha}} & , R(\alpha) > 0 \\ 1 & , R(\alpha) = 0 \\ \int_a^t (d\tau)^{-\alpha} & , R(\alpha) < 0 \end{cases} \quad (1)$$

Where, $a$denotes lower limit of integration, $\alpha(\alpha \in R)$ represents the order of fractional differentiation or integration, and $R(\alpha)$ denotes the real part of α.

2.2. Oustaloup approximation method

The Oustaloup approach rests on the function approximation derived from as [18]:

$$G_f(s) = s^{\alpha}, \alpha \in R^{+} \quad (2)$$

With consideration for the rational function:

$$G_f(s) = K \prod_{k=1}^{N} \frac{s+w'_k}{s+w_k} \quad (3)$$

Still, the poles, zeros, and gain are assessable as:

$$w'_k = w_b . w_u^{(2k-1-\gamma)/N}, w_k = w_b . w_u^{(2k-1+\gamma)/N}, K = w_h^{\gamma}$$

In a geometrically dispersed frequency band, w_u denotes the central frequency and the unity gain in frequency. Let $w_u = \sqrt{w_h w_b}$, : w_h and $w_b$respectively reflect the upper and lower frequencies. The orders of derivative and filter respectively are γ and N.

2.3. Reduction of fractional-order models

Model reduction involves approximating a high-order model with a lower-order counterpart. The use of filters to approximate fractional-order operators often leads to high-order transfer functions, necessitating a reduction. It is essential to define an optimization criterion in order to achieve the most efficient possible reduced-order model. The error is subsequently used to construct an objective function. The objective function transforms the problem into a numerical optimization challenge. This issue can only be addressed using numerical optimization methods, and the approach proposed in references [22], [23] and[24] provides an optimal algorithm for model reduction by following these steps:

- **Step 01**: Provide the high-order model.
- **Step 02**: Select the desired orders for the numerator and denominator of the reduced-order model.
- **Step 03**: Define a function that describes the objective function.
- **Step 04**: Utilize a solver to determine the optimal reduced-order model.

3. Modeling of DC Motor

The model of the DC Motor is shown in Figure 1. The position $q(t)$, the controlled variable, will be governed by the applied voltage V_a.

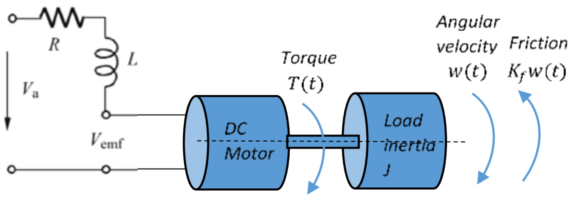

Figure 1.General model of a DC Motor.

The angular velocity $w(t)$ is the controlled variable for speed control, and the transfer function takes the following form: [23], [24].

$$G_{DC-motor}(s) = \frac{w(s)}{V_a(s)} = \frac{K_m}{(L_a s+R_a)(Js+b)+K_b K_m} \quad (4)$$

But for a lot of DC motors, the armature's time constant $\tau_a = \frac{L_a}{R_a}$ is insignificant, hence the model can be reduced to:

$$G_{DC-motor}(s) = \frac{w(s)}{V_a(s)} = \frac{K_m}{R_a(Js+b)+K_b K_m} = \frac{\frac{K_m}{R_a b+K_b K_m}}{\tau s+1} = \frac{K_{DC-motor}}{\tau s+1} \quad (5)$$

where $\tau = \frac{R_a J}{R_a b+K_b K_m}andK_{DC-motor} = \frac{K_m}{R_a b+K_b K_m}$

In this work we use the identified model of DC Motor [25]:

$$G_{DC-motor}(s) = \frac{w(s)}{V_a(s)} = \frac{K_{DC-motor}}{\tau s+1} = \frac{0.25}{1.45s+1} \quad (6)$$

with the following parameters values:
$K_{DC-motor}$=0.25, $\tau = 1.45$

4. Optimization by Genetic Algorithms

Natural selection and genetics are the foundations of the Genetic Algorithm, a bioinspired optimization method [23], [24]. Through the evolution of a population of potential solutions over multiple generations, it seeks to identify the best answer to an issue. Each stage is explained in full below:

Step 1: Initialization

- ***Objective***: The goal function $f(x)$ that needs to be optimized should be defined.
- ***Generate the initial population****:* Make an initial population of N individuals (potential solutions), each of whom is represented by a chromosome (a collection of parameters that have been encoded). A binary string, actual values, or any other representation appropriate for the issue can be used as the chromosome.
- **Set parameters**: Define the population size (N), number of generations ($MaxGen$), crossover probability (p_c), mutation probability (p_m), and other relevant parameters.

Step 2: Evaluate Fitness

- Evaluate the fitness of each individual in the population using the objective function $f(x)$.
- The fitness value indicates how good each candidate solution is.

Step 3: Selection

Based on their fitness, choose parents from the existing population. Better solutions have a higher chance of being chosen in this typically stochastic process.

Step 4: Crossover (Recombination)

To create new offspring (children), perform crossover between chosen parent individuals.

Step 5: Mutation

Apply mutation to progeny with a p_m probability. The purpose of mutation is to preserve genetic variety and avoid premature convergence by introducing random changes to specific genes inside the chromosome.

Step 6: Evaluate New Population

- Calculate the fitness of the new offspring population.
- Combine the offspring with the current population if needed, depending on the chosen GA strategy.

Step 7: Replacement

- Select individuals for the next generation based on fitness.

Step 8: Check Stopping Criterion

The algorithm should be terminated if the maximum number of generations (Max Gen) is reached or if the fitness improvement falls below a predetermined threshold.

Step 9: Return the Best Solution

- The best individual from the final population should be returned as the optimal solution.

5. Control Strategy

5.1. Adaptive integer PID controller

The integer APID feedback control law is given by equation (7) [16].

$$u(t) = -k_c[k_1(t)e(t) + I\{k_2(t)e(t)\} + D(k_3(t)e(t))] \quad (7)$$

With:

$$k_1(t) = k_p(t) + \alpha_1 k_i(t) + \alpha_3 k_d(t)$$

$$k_2(t) = \alpha_2 k_i(t), k_3(t) = \alpha_4 k_i(t), k_p(t) = e^2(t) \quad (8)$$

$$k_i(t) = I\{e^2(t)\} \text{and } k_d(t) = D\{e^2(t)\}$$

$$e(t) = y(t) - r(t)$$

where: Kp is the proportional gain, Ki is the integral time constant, Kd is the derivative time constant, and k_c, α_1 *and* α_2 are positive constants.

Figure2 shows a classical adaptive PID control system for a DC motor, enhanced by a genetic algorithm. The desired output (y) is compared to the actual output (r) to calculate the error (e), which is sent to the PID controller. The PID gains (K_p, K_i, K_d) are optimized using a genetic algorithm to improve performance. The genetic algorithm evaluates an objective function that measures system performance and adjusts the gains for optimal control. The control signal (u) from the PID controller drives the DC motor, and the actual output is fed back to form a closed-loop system. This approach ensures precise and adaptive control of the motor.

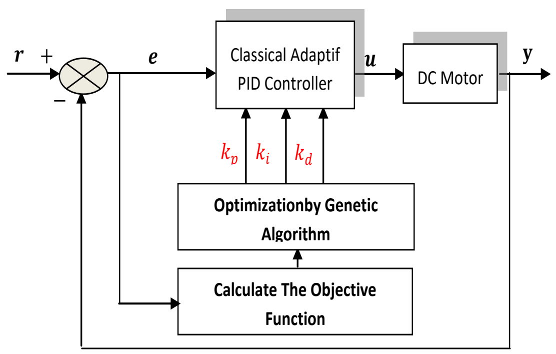

Figure 2. Classical adaptive PID control of DC motor.

5.2. Fractional adaptive PI$^\lambda$D$^\mu$ controller

Equation (9) presents the FAPID feedback control law.

$$u(t) = -k_c[k_1(t)e(t) + I^\lambda\{k_2(t)e(t)\} + D^\mu(k_3(t)e(t))] \quad (9)$$

with:

$$k_1(t) = k_p(t) + \alpha_1 k_i(t) + \alpha_3 k_d(t)$$

$$k_2(t) = \alpha_2 k_i(t) \quad (10)$$

$$k_3(t) = \alpha_4 k_i(t)$$

$$k_p(t) = e^2(t)$$

$$k_i(t) = I^\lambda\{e^2(t)\}$$

$$k_d(t) = D^\mu\{e^2(t)\}$$

$$e(t) = y(t) - r(t)$$

where: λ is an integrate operator and μ is derivative operator.

Figure 3 shows a control system using a Fractional Adaptive PID Controller that has been tuned using a Genetic Algorithm. The controller controls a DC motor and adjusts its parameters $(K_p, K_i, K_d, \lambda, \mu)$ to minimize an objective function depending on system performance. The procedure includes a feedback loop in which the motor output is compared to a reference input.

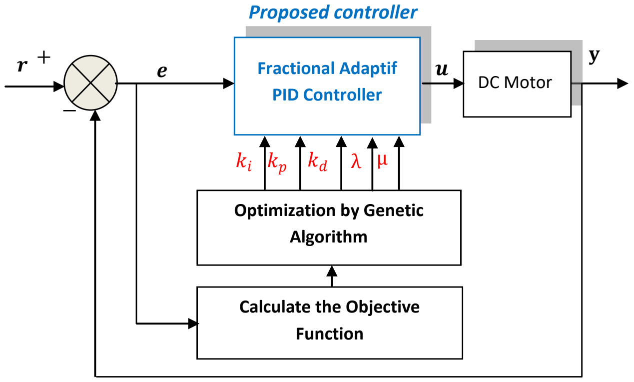

Figure3. Fractional adaptive PID control system.

6. Results and Discussion

The fitness function F, chosen for minimization, is defined using the Mean Absolute Error (MAE). This metric quantifies the difference between the measured output (y) and the desired output (r), as given by the formula:

$$MAE = \frac{1}{N}\sum_{i=1}^{N} |y(i) - r(i)| \quad (11)$$

Here, y represents the system's measured output, and r is the reference input.

Figure 4 illustrates the speed response of the DC motor controlled by APID Controller, with the optimized parameters as follows:

- $k_p = 966.6787$,
- $k_i = 426.8123$,
- $k_d = 48.3883$.

As can be seen from Figure 4, the top graph illustrates the speed response of the DC motor controlled by APID Controller, where the motor quickly reaches the desired speed with no overshoot and stabilizes within 0.2 seconds. Meanwhile, the bottom graph depicts the fitness values during the 100 iterations of the optimization process, with blue points representing various solutions and red markers indicating the optimal values, demonstrating successful convergence of the optimization.

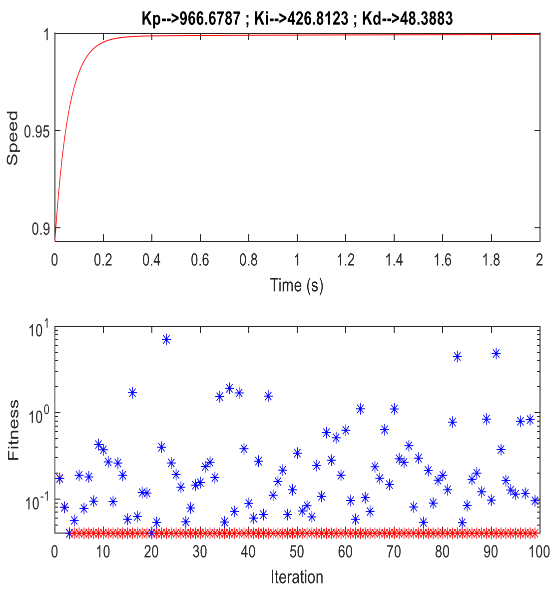

Figure4. Speed DC motor using the integer adaptive PID controller.

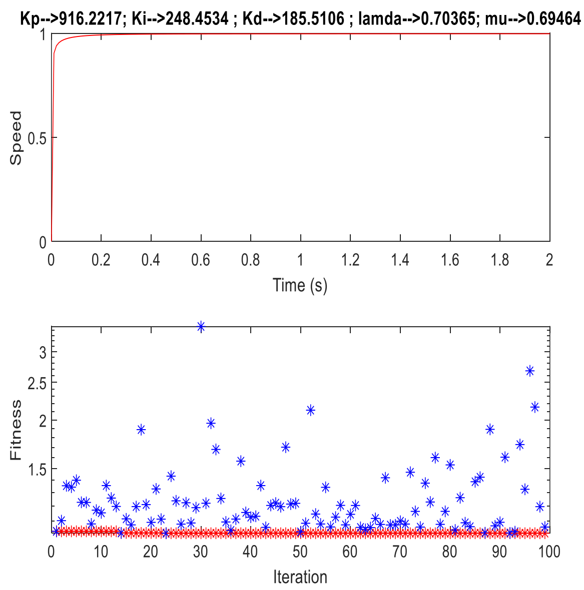

Figure5. Speed DC motor using the fractional adaptive PID controller.

Figure 5 shows the speed of DC Motor using the FAPID Controller with the following optimized parameters values:

- $k_p = 916.2217$,
- $k_i = 248.4534$,
- $k_d = 185.5106$,
- $\lambda = 0.70365$,
- $\mu = 0.69464$.

where,$\lambda$is a parameter that influences the order of integration in the fractional controller and μsi a parameter that affects the order of differentiation in the fractional controller.As can be seen from the Figure 5, the top plot depicts the speed response of a DC motor controlled by a fractional adaptive PID controller. The motor immediately achieves the necessary speed with minimal overshoot or oscillations, displaying great stability, accuracy, and rapid settling time. The bottom figure depicts the optimization of controller settings using a metaheuristic method. The blue dots reflect the fitness values of candidate solutions at each iteration, while the red curve denotes the optimal fitness. Initially, the algorithm tests a broad range of solutions (high variability in fitness), but as iterations go, the fitness values converge, indicating efficient parameter tuning. The relationship between the two figures demonstrates how the adjusted controller settings enable the smooth and speedy speed response shown in the top plot.

Table 1 compares the transient response stability metrics of a DC motor system with two different control schemes: the standard APID controller and the suggested FAPID controller.

Table 1. Transient Response Stability Parameters of DC Motor System

Controllers	Overshoot [%]	Setling time [s]	Rise time [s]	Mean Absolute Error (rad/s)
APID	0.0000	0.2479	0.1298	0.0086
FAPID	0.0000	0.0683	0.0084	0.0032

- **For the over shoot [%]:** Both controllers have 0% overshoot, preventing undesired oscillations over the required speed. This demonstrates how both strategies provide a steady and well-damped response.
- **For the settling time [s]:** The proposed FAPID controller exhibits a quicker settling time of 0.0683 seconds than the APID controller, which takes 0.2479 seconds. This demonstrates the FAPID's improved capacity to get the system to a steady state more rapidly, which is crucial in real-time applications that need rapid reaction.
- **For the rise time [s]:** The proposed FAPID controller has a substantially lower rise time (0.0084 seconds) compared to the APID controller (0.1298 seconds), indicating faster initial response to step

inputs. This highlights how the FAPID controller can provide a substantially quicker initial reaction, making it more successful in dynamic conditions.

- **For Mean Absolute Error (MAE) [rad/s]:** The proposed FAPID controller has a reduced MAE of 0.0032 rad/s compared to 0.0086 rad/s for the APID controller. This shows that the FAPID tracks the required speed more accurately, reducing deviations throughout the process.

In conclusion, the FAPID controller's significant advancements show its potential for applications demanding accurate and quick control, such as robotics, automotive systems, and industrial automation. The use of fractional calculus not only improves transient responsiveness, but it also allows more flexibility and adaptability when tweaking the controller. This makes the FAPID controller a better option than previous techniques for assuring optimum performance in dynamic and complicated contexts.

7. Conclusion

This study provided a genetic algorithm-optimized FAPID controller to improve the performance of a DC motor. The findings showed that the FAPID controller beat the classic APID controller in many important performance parameters, including rising time, settling time, overshoot, and mean absolute error. These advantages are ascribed to fractional calculus's increased flexibility and accuracy, which allows for better modeling and management of the system's complicated dynamics.

In industrial process control, FOAPID controllers are growing in popularity, particularly for systems with inaccurate models or uncertainties. They are appropriate for robotics, HVAC systems, electrical systems, and chemical processes due to their capacity to manage changing system dynamics. Compared to traditional APID, the FAPID approach significantly enhances noise rejection and provides greater flexibility in managing complex transient responses. By incorporating fractional integrators and derivatives, the fractional PID controller allows for finer system tuning and adaptability to varying conditions, making it particularly effective for complex and precision-dependent applications. This improved response capability makes FAPID a more robust solution for modern control environments, where traditional APID may struggle with noise and precision limitations.

The results have important implications for industrial applications that need fast, precise, and dependable responses, such as robots, automotive systems, and automation. Furthermore, this technique has the potential to be applied to other control systems, laying the groundwork for optimization in a variety of disciplines.

Research is being done on FOAPID controllers to increase the efficiency and stability of energy systems, including integration with power distribution networks and smart grids as in automation and robotics where FOAPID controllers are used to increase the accuracy and resilience of motion control, especially for systems that have complicated dynamics or need to react very flexibly to outside disturbances.

Finally, this paper emphasizes the expanding relevance of fractional calculus in the area of automated control, presenting a viable option for improving existing approaches. Furthermore, future research will look at how the adaptive control technique might be applied to systems with partial or distributed dynamics to increase resilience, noise suppression, and overall performance.

Conflicts Interest Statement

The authors confirm that they do not have any competing financial interests in this paper.

Data Availability Statement

All data produced or tested in this study are integrated into this article.

References

[1] C. A. Monje, Y. Chen, B. M. Vinagre, D. Xue, and V. Feliu-Batlle, “Fractional-order systems and controls: fundamentals and applications,” in *Springer Science & Business Media,* Berlin, Germany: Springer, 2010, pp. 1–615.

[2] M. Nesri, H. Benkadi, K. Nounou, G. Sifelislam, and M. F. Benkhoris, "Fault tolerant control of a dual star induction machine drive system using hybrid fractional controller," *Power Electronics and Drives,* vol. 9, no. 1, pp. 161–175, 2024.

[3] S. Guedida, B. Tabbache, K. Nounou, and A. Idir, “Reduced-Order Fractionalized Controller for Disturbance Compensation Based on Direct Torque Control of DSIM With Less Harmonic,” *ELECTRICA*, vol. 24, no. 2, pp. 450–462, 2024.

[4] A. Idir, H. Akroum, S. A. Tadjer, and L. Canale, “A comparative study of integer order PID, fractionalized order PID and fractional order PID controllers on a class of stable system,” in *Proceedings of the 2023 IEEE International Conference on Environment and Electrical Engineering and 2023 IEEE Industrial and Commercial Power Systems Europe (EEEIC/I&CPS Europe)*, pp. 1–6, June 2023.

[5] I. Podlubny, Fractional Differential Equations. *Mathe matics in Science and Engineering. Academic Press Inc.*, San Diego, CA, Vol. 198, 1999.

[6] Y. Bensafia, A. Idir, K. Khettab, M. S. Akhtar, and S. Zahra, "Novel robust control using a fractional adaptive PID regulator for an unstable system," *Indonesian Journal of Electrical Engineering and Informatics (IJEEI),* vol. 10, no. 4, pp. 849–857, 2022.

[7] S. Ladaci, J. J. Loiseau, and A. Charef, "Adaptive Internal Model Control with fractional order parameter," *International Journal of Adaptive Control and Signal Processing,* vol. 24, pp. 944–960, 2010.

[8] A. Eltayeb, G. Ahmed, I. H. Imran, N. M. Alyazidi, and A. Abubaker, "Comparative Analysis: Fractional PID vs. PID Controllers for Robotic Arm Using Genetic Algorithm Optimization," *Automation,* vol. 5, no. 3, pp. 230–245, 2024.

[9] R. Kiruba and K. Malarvizhi, "Fractional PID with Genetic Algorithm Approach for Industrial Tank Level Control Process," *Electric Power Components and Systems,* pp. 1–15, 2024.

[10] A. Idir, Y. Bensafia, and L. Canale, “Influence of approximation methods on the design of the novel low-order fractionalized PID controller for aircraft system,” *Journal of the Brazilian Society of Mechanical Sciences and Engineering,* vol. 46, no. 2, pp. 1-16, 2024.

[11] W. V. Balasaheb and C. Uttam, "An intelligent optimized fractional order sliding mode controller for biological system," *Multimedia Tools and Applications,* pp. 1–22, 2024.

[12] Y. Bensafia, K. Khettab, and A. Idir, “A novel fractionalized PID controller using the sub-optimal approximation of FOTF,” *Algerian Journal of Signals and Systems*, vol. 7, no. 1, pp. 21–26, 2022.

[13] Y. Bensafia, K. Khettab, and A. Idir,"An Improved Robust Fractionalized PID Controller for a Class of Fractional-Order Systems with Measurement Noise," *International Journal of Intelligent Engineering and Systems,* vol. 11, no. 2, pp. 200–207, 2018.

[14] Y. Chen and K. L. Moore, "Fractional-order control: A tutorial," in *Proceedings of the American Control Conference,* vol. 6, pp. 4985–4990, 2002.

[15] K. Khettab, S. Ladaci, and Y. Bensafia, "Chattering Elimination in Fuzzy Sliding Mode Control of Fractional Chaotic Systems Using a Fractional Adaptive Proportional Integral Controller," *International Journal of Intelligent Engineering and Systems,* vol. 10, no. 5, pp. 255–265, 2017.

[16] K. Haneet, S. Parul, and A. Pawanesh, "Analysis of fitness function in genetic algorithms," *Journal of Scientific and Technical Advancements,* vol. 1, no. 3, pp. 87–89, 2015.

[17] A. Oustaloup, F. Levron, B. Mathieu, and F. Nanot, "Frequency-Band Complex Noninteger Differentiator: Characterization and Synthesis," *IEEE Transactions on Circuits and Systems I,* vol. 47, no. 1, pp. 25–39, 2000.

[18] A. Oustaloup, J. Sabatier, and P. Lanusse, "From fractal robustness to CRONE control," *Fractional Calculus and Applied Analysis,* pp. 1–30, 1999.

[19] J. Sabatier, A. Oustaloup, A. G. Iturricha, and F. Levron, "CRONE control of continuous linear time periodic systems: Application to a testing bench," *ISA Transactions,* pp. 421–436, 2003.

[20] S. Dadras and H. R. Momeni, "Control of a fractional-order economical system," *Physica A,* vol. 389, pp. 2434–2442, 2010.

[21] A. Idir, L. Canale, Y. Bensafia, and K. Khettab, "Design and robust performance analysis of low-order approximation of fractional PID controller based on an IABC algorithm for an automatic voltage regulator system," *Energies,* vol. 15, no. 23, p. 8973, 2022.

[22] A. Idir, Y. Bensafia, K. Khettab, and L. Canale, "Performance improvement of aircraft pitch angle control using a new reduced order fractionalized PID controller," *Asian Journal of Control,* vol. 25, no. 4, pp. 2588–2603, 2023.

[23] X. Liu, Y. Wang, and J. Zhang, "Fractional-Order PID Control for DC Motors Based on Genetic Algorithm Optimization," *IEEE Access,* vol. 11, pp. 12914–12922, 2023.

[24] S. K. S. K. T. R. Mandal and S. K. Saha, "Design and Analysis of a Fractional Adaptive PID Controller for DC Motor Control Using Genetic Algorithms," *Mathematical Problems in Engineering,* Article ID 4671623, 2022.

[25] Y. Bensafia, K. Khettab, and S. Ladaci, "DC-Motor Velocity Control Using a Robust Fractionalized Adaptive PI Controller," in *16th International Conference on Sciences and Techniques of Automatic Control & Computer Engineering (STA'2015),* Monastir, Tunisia, Dec. 21–23, 2015.