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Reducing the Delay Time and Tracking Trajectory of the Robot SCARA Using the Fractional PID Controller

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Abstract

In recent years, there has been a significant increase in the study of fractional systems and fractional-order control (FOC), which has proven effective in enhancing plant dynamics, particularly in terms of disturbance rejection and response time improvement. Traditionally, the Proportional-Integral-Derivative (PID) controller has been valued for its simplicity and ease of parameter adjustment. However, as the complexity of control systems escalates, several specialised PID controllers have been designed to address specific challenges. Despite its effectiveness, the conventional PID controller often faces limitations in complex systems requiring high precision and adaptive dynamics. Researchers have increasingly focused on the Fractional Proportional-Integral-Derivative (FPID) controller to address these deficiencies. The FPID controller incorporates fractional integrators and derivatives, facilitating improved tuning of system dynamics and offering increased control over response characteristics. This study introduces a fractional integrator in PID control to improve trajectory tracking and reduce delay time in Selective Compliance Articulated Robot Arm (SCARA) systems. Unlike traditional PID controllers, which may struggle with high-frequency noise and parameter variations, the fractional integrator offers enhanced noise suppression and adaptability. The fractional PID approach is relevant beyond robotics, including many systems like temperature control, electrical motor regulation, power electronics, and biomedical control systems, where accuracy and resilience to disturbances are essential. Unlike traditional PID, the proposed technique offers more adaptability in handling transient responses and greater disturbance suppression, making it a viable solution for modern, complex control environments.

***Keywords**:* Fractional Control (FC), Approximation Methods, Selective Compliance Articulated Robot Arm (SCARA), Performance Analysis.

1. Introduction

In recent years, the use of fractional-order calculus has been significant in the modelling and development of controllers for dynamic systems [1], [2], [3], [4]. This increased focus originates from the enhanced ability of fractional-order calculus to more accurately represent real systems in comparison to integer-order calculus [5]. Various methods for approximating fractional-order calculus have enabled its use in other domains, including control theory and electrical circuit theory [6].

The primary benefit of FOC is its robustness. The inaugural documented FOC system in the literature, introduced around twenty years ago, is designated as the "Commande Robuste d'Ordre Non Entier" (CRONE) controller [3]. This controller employs the constant phase

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characteristic of the ideal Bode transfer function $1/s^{\alpha}$ to provide robust feedback control despite variations in gain. Studies also show that fractional-order systems, which are known as feedback control processes with long memory, make the system more responsive to changes and better able to handle noise and other disturbances [7], [8], [9].

Robotics has undergone significant evolution since the inception of the field, marked by the development of the pioneering industrial robot by Griffith P. Taylor in 1937 [10]. This important step forward made it possible for robotic control systems to get better, like the fractional PID controllers we looked at in this study. These controllers are meant to help modern multi-joint robotic applications that need more precision and stability.

Many studies have focused on developing new robust fractional-order controllers [11], [12], [13], [14], [15], [16], with most approaches based on the CRONE methodology [17]. In [8], Caponetto et al. propose an innovative FOC design employing a robust tuning approach for PID control, followed by further advancements in [18]. Additionally, the works in [19] and [20] present a fractional integral version of the PI/PID controller.

Shah and Agashe [21] provide a comprehensive overview of fractional PID controllers, highlighting their flexibility and stability in complex systems. Our approach similarly leverages fractional-order calculus for improved tracking accuracy and stability but specifically addresses the issue of temporal lag in trajectory tracking, an aspect less emphasized in general reviews of fractional PID control. In another related work, Djeffal et al. [22] introduce an optimized torque control method for continuum robots, focusing on dynamic model enhancement and precise torque control through advanced optimization techniques. Although our study does not address continuum robots directly, our fractional-order PID (FPID) controller shares the goal of precision enhancement, extending this to multi-joint robotic systems with an emphasis on coordination and phase alignment across joints. This complements the OCTC approach, as both methods seek to reduce response lag and enhance accuracy, though applied to different robotic setups.

Similarly, Tanyıldızı's [23] work on fractional PID control for exoskeletons demonstrates adaptation to human motion and improved stability in dynamic environments. Our FPID approach also seeks to enhance tracking precision but is tailored to multi-joint robotic systems following complex trajectories, with a unique focus on synchronizing movements across joints. This attention to multi-joint coordination and phase alignment is especially relevant in SCARA robots, where precise inter-joint timing is critical.

This study primarily focuses on applying a fractional PID controller to minimize delay and accurately track both the robot's trajectory and the designated reference path. This is accomplished by incorporating fractional-order filters into the traditional PID feedback loop.

This paper is organized as follows: Section 2 covers the fundamentals of fractional-order systems. Sections 3 and 4 present the integer and fractional PID controllers, respectively. Section 5 discusses the application of the fractional PID controller to the robot, with simulation results provided in Section 6. Finally, the conclusion and directions for future works are presented in Section 7.

2. Fractional Calculus Fundamentals

2.1. Definition

Fractional calculus, which emerged in the 17th century, generalizes the notions of derivatives and integrals to non-integer orders, facilitating the assessment of integrals when the order nnn may be fractional, irrational, or complex. This domain has swiftly broadened in applications, providing a more precise depiction of real-world phenomena compared to conventional approaches. Currently, methods for estimating fractional derivatives and integrals render fractional calculus significant in control theory, electrical circuit theory, and capacitor theory [1], [2], [10], [18].

The operator that unifies differentiation and integration is expressed as follows:

$$aD_t^q = \begin{cases} \frac{d^q}{dt^q} & ,R(q) > 0 \\ 1 & ,R(q) = 0 \\ \int_a^t (d\tau)^{-q} & ,R(q) < 0 \end{cases} \quad (1)$$

where,

$R(q)$ represents the real part of the fractional order q. The shown generalized fundamental operator equation incorporates differentiation and integration based on the value of $R(q)$.

Domain of $R(q)$:

- $R(q) > 0$: The operator represents fractional differentiation of order q.
- $R(q) = 0$: The operator represents the identity (or constant) operation, essentially performing no differentiation or integration.
- $R(q) < 0$: The operator represents fractional integration of order $-q$ [2].

a – Integration lower limit.

t – Integration upper limit.

Developed primarily in the 19th century, fractional-order derivative theory includes several definitions. The Grunwald-Letnikov (GL) definition, widely recognized and valued for its applicability in discrete control algorithms [5], [8], [10], is formulated as follows:

$$D^{\alpha} f(t) = \lim_{h \to 0} h^{-\alpha} \sum_{j=0}^{k} (-1)^j \binom{\alpha}{j} f(kh - jh) \quad (2)$$

Where the coefficients are evaluated from:

$$\omega_j^{(\alpha)} = \binom{\alpha}{j} = \frac{\Gamma(\alpha + 1)}{\Gamma(j + 1)\Gamma(\alpha - j + 1)}$$

and h is the step time .

The Riemann-Liouville (RL) definition is formulated as:

$$f(t) = \frac{d^n}{dt^n} \int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \quad (3)$$

For numerous functions relevant to real-world physical and engineering contexts, the RL and GL formulations are comparable [2].

2.2. Fractional order systems

Feedback control systems are a key area where fractional calculus is applied to improve efficiency, robustness, and flexibility. For the control engineer, fractional calculus helps compensate for shifts in the transfer function due to parametric variations, aging, and other factors [17].

The fractional order system is given as:

$$G(s) = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \cdots + b_1 s^{\beta_1} + b_0}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \cdots + a_1 s^{\alpha_1} + a_0} \quad (4)$$

where a_i and b_j are real numbers such that

$$\begin{cases} 0 \le \alpha_0 \le \alpha_1 \le \cdots \le \alpha_n \\ 0 \le \beta_0 \le \beta_1 \le \cdots \le \beta_m \end{cases}$$

and $\boldsymbol{s}$ is the Laplace operator.

The Oustaloup approximation is a generalized differential action derivator that covers the frequency spectrum, ensuring minimal phase behavior and reducing differential behavior within a defined restricted frequency range based on application needs.

The approach relies on function approximation derived from:

$$H(s) = S^{\alpha}, \alpha \in R^{+} \quad (5)$$

The function given in eq 6, represents a rational function approximation of a fractional-order differential operator. This is often used in control systems and fractional calculus when implementing fractional-order derivatives, as they are not Integer-Order operators and can't be represented directly in a transfer function.

Podlubny's in [3] provides a practical way to approximate fractional derivatives by creating a rational transfer function composed of real poles and zeros, which cover a certain frequency range. The purpose of this method is to approximate the behavior of a fractional derivative across the desired frequency range, thereby allowing its implementation using Integer-Order transfer functions, which are compatible with conventional control design techniques.

$$G_f(s) = K \prod_{k=1}^{N} \frac{s + w_k'}{s + w_k} \quad (6)$$

The zeros, poles and gain are obtained from:

$$w_k' = w_b . w_u^{\frac{2k-1-\gamma}{N}}, w_k = w_b . w_u^{\frac{2k-1+\gamma}{N}}, K = w_h^{\gamma}$$

Where w_u It represents the unity gain frequencies and the central frequency of a geometrically distributed frequency band. Let $w_u = \sqrt{w_h w_b}$, where w_h and w_b are respectively the upper and lower frequencies. γ is the order of derivative, and N is the order of the filter.

3. Controllers Design

3.1. Integer order PID controller

Figure 1 illustrates the feedback control loop of an integer order system, depicted as:

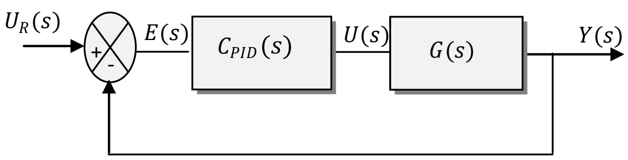

Figure 1. Feedback control loop of integer PID controller.

Where, $U_R(s)$ represents the input signal, $E(s)$ denotes the error signal, $C_{PID}(s)$ refers to classical PID controller's transfer function, $G(s)$ represents the plant transfer function, $Y(s)$ denotes the output signal, and $U(s)$ corresponds to the controller signal.

Equation 7 represents the transfer function of a Proportional-Integral-Derivative (PID) controller in the Laplace domain, which is commonly used in control systems to adjust the output of a system to reach a desired setpoint.

$$C(s) = K_p(1 + \frac{1}{T_i s} + T_d s) \quad (7)$$

where:

- K_p is the proportional gain, which determines the response based on the current error. Increasing K_p results in a stronger reaction to the current error but can lead to overshoot or oscillations if too high.
- T_i is the integral time constant. The term $\frac{1}{T_i s}$ represents the integral action, which considers the accumulation of past errors, helping to eliminate steady-state errors by gradually adjusting the output.
- T_d is the derivative time constant. The term $T_d s$ represents the derivative action, which considers the rate of change of the error, helping to dampen the response and reduce overshoot by reacting to the speed of the error change.

3.2. Fractional order PID controller

The structure of the feedback control loop for a fractional integer order system is illustrated in Figure 2 as follows:

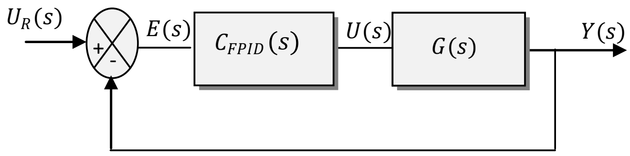

Figure 2. Feedback control loop of fractional PID controller.

Where, $C_{FPID}(s)$ is Fractional Controller Transfer Function. Equation 8 represents the transfer function of a Fractional-Order PID (FOPID) controller. Fractional-order controllers extend the standard Integer-Order PID controllers by allowing non-integer (fractional) orders for the integral term only.

$$C_{FPID}(s) = K_p \left(1 + \frac{1}{T_i s^{\alpha}} + T_d s\right) \quad (8)$$

where:

- K_p is the proportional gain, similar to the standard PID controller.
- T_i is the integral time constant, but here it's associated with ss^{α} instead of just s. The term $T_i s^{\alpha}$ introduces a fractional-order integral action.
- T_d is the derivative time constant, same as in the Integer-Order PID, associated with the derivative term $T_d s$.
- α is a fractional order (where $0 < \alpha < 1$), allowing more flexibility in tuning the integral action.

4. Fractional PID Controller (FPID) of the Robot

The proposed global control approach for managing the robot relies on position feedback, as shown in Figure 3. This system incorporates an outside control loop and an inner control loop, each fulfilling distinct functions to guarantee accurate and steady placement.

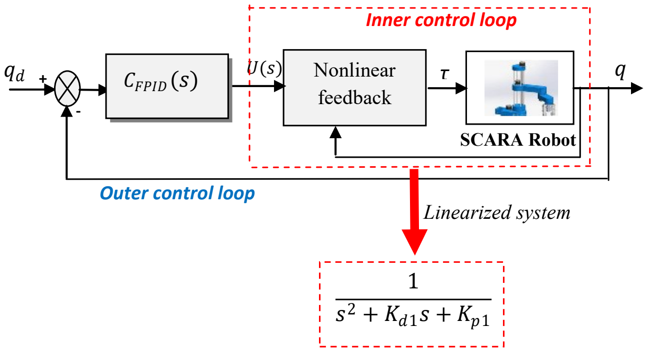

Figure 3. Control scheme for robot control using the FPID approach.

1. Outer Control Loop:

- The outer loop is responsible for overall stability and robustness of the control system. It begins with a reference signal (q_d), representing the desired position or set-point for the robot.

- The reference signal is handled by a fractional proportional-integral-derivative (FPID) controller, C_{FPID}, which produces the control input U required to direct the system toward the desired position.

- The output of the outer control loop, U, is passed to the inner control loop, which regulates the dynamics more intricately connected to the physical structure of the robot.

2. Inner Control Loop:

- The inner loop is responsible for regulating the nonlinear behaviors and dynamic responsiveness of the SCARA robot.

- A nonlinear feedback system is utilized to mitigate the robot's nonlinearities, efficiently adjusting for intrinsic complexity such as joint friction, link flexibility, and other mechanical flaws.

- The SCARA robot employs the control signal from the outer loop and, through nonlinear feedback modifications, achieves the target position (q).

3. Linearized System Model:

- The system's behavior can be depicted using a linearized model, as seen in the figure, to facilitate study and enhance clarity. The transfer function $\frac{1}{s^2+K_{d1}s+K_{p1}}$ represents a second-order approximation, where K_d is the velocity gain and K_p is the position gain.

This dual-loop control technique ensures precise placement of the SCARA robot by integrating an FPID-based outer loop with a nonlinear feedback mechanism in the inner loop, hence maintaining stability and responsiveness in the presence of nonlinearities. The use of a linearized model enables the optimization of control parameters to attain defined performance objectives, including the reduction of overshoot, minimization of settling time, and elimination of steady-state error.

4.1. Dynamic model of the robot

This dynamic model can be written in the following matrix form [6], [24], [25]:

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \quad (9)$$

Where

$B(q) \in R^3 \times R^3$: is inertia matrix,

$C(q,\dot{q}) \in R^3$: Represents the matrix of centrifugal and Coriolis components.

$G(q) \in R^3$: Denotes the vector of gravitational forces.

$\tau \in R^3$: Represents the control inputs vector (torques applied by the actuators).

$q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \in R^3$ represents the vector of joint locations,

$\dot{q} = \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{pmatrix} \in R^3$ is the articular velocities vector,

$\ddot{q} = \begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{pmatrix} \in R^3$ is the articular accelerations vector,

4.2. SCARA robot linearized model

The robot's linearized model, incorporating nonlinear feedback, is represented by the subsequent equivalent transfer function:

$$G(s) = \frac{1}{s^2+K_{d1}s+K_{p1}} \quad (10)$$

where K_{p1} represents the position gain and and K_{d1} represents the velocity gain.

The SCARA robot parameters are enumerated in Table 1.

Table 1. SCARA Robot parameters.

Segment	Segment 0	Segment1	Segment2	Segment3
Mass $[Kg]$	m0= 19:5	m1 = 8	m2 = 6	m3 = 0:5
Length $[m]$	d0 = 0:65	d1 = 0:4	d2 = 0:3	d3 = 0:3
Inertia [kg.m^2]	I0= 1.0298	I1 = 0.16	I2= 0.0675	I3= 0.0056

The reference trajectory is:

$$q_1 = q_2 = q_3 = \frac{10\pi}{180} \sin(2\pi t) \quad (11)$$

The parameters of the robot's linearized model are:

$$K_{p1} = 2500 \text{ and } K_{d1} = 500.$$

5. Numerical Simulations

This section provides the simulation results for the proposed architecture wich integrates a linearizing feedback and the proposed FPID controller.

The parameters of PID and FPID controllers are:

$$K = 10, T_i = 1.5, T_I = 80 \text{ and } \alpha = 0.5.$$

Figures 4 and 5 show the articular reference trajectory and the trajectories using the PID and fractional PID (FPID) controllers, respectively.

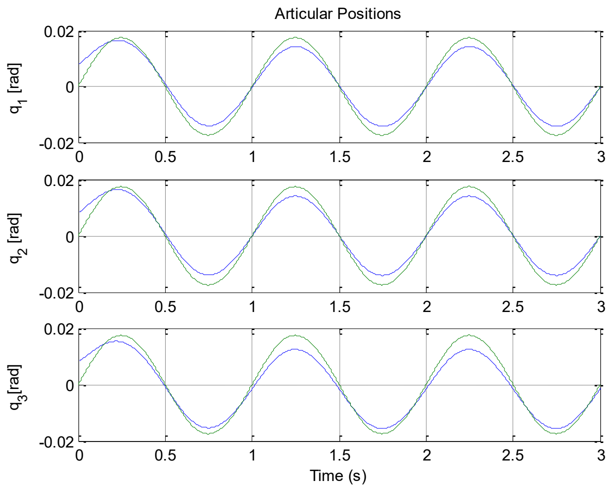

Figure 4. Comparison of articular reference trajectory and PID controlled trajectory.

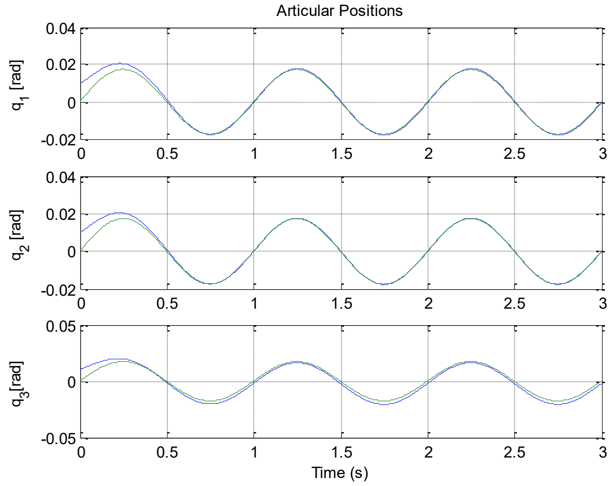

Figure 5. Comparison of articular reference trajectory and FPID controlled trajectory.

In comparing Figure 4 (PID approach) and Figure 5 (FPID approach) for the articular reference trajectory:

- ***Tracking Accuracy***: In Figure 5 (FPID approach), the trajectory follows the reference trajectory (green line) more closely than in Figure 4 (PID approach). The FPID approach shows reduced tracking error, especially in the mid-sections of each oscillation cycle, indicating improved alignment with the desired trajectory.
- ***Oscillation and Stability***: The PID approach in Figure 4 exhibits slight deviations in oscillatory response, with visible differences in amplitude and phase lag. The FPID approach in Figure 5 demonstrates a more stable response with less overshoot and a smaller phase difference, suggesting enhanced robustness and stability.
- ***Amplitude and Phase***: The FPID controller better maintains the amplitude of the desired trajectory across the graphs, while the PID controller shows slight discrepancies, especially at the peaks and troughs of each cycle.

Overall, the FPID approach (Figure 5) provides smoother and more accurate trajectory tracking compared to the traditional PID approach in Figure 4, reflecting the advantages of fractional-order control in handling complex dynamics.

Figures 6 and 7 show the articular speed reference trajectory and the trajectories using the PID and fractional PID (FPID) controllers, respectively.

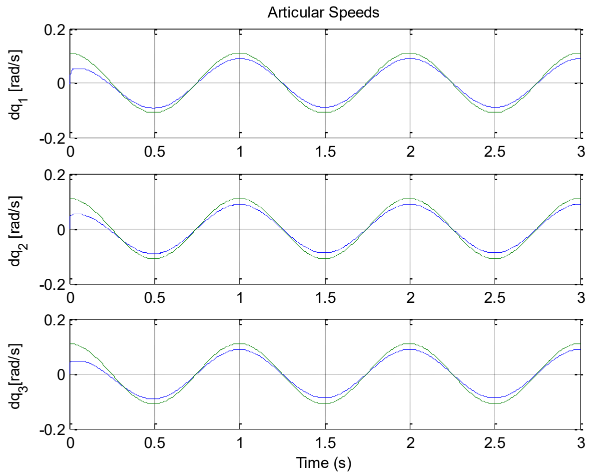

Figure 6. Comparison of articular reference speed trajectory and PID controlled trajectory.

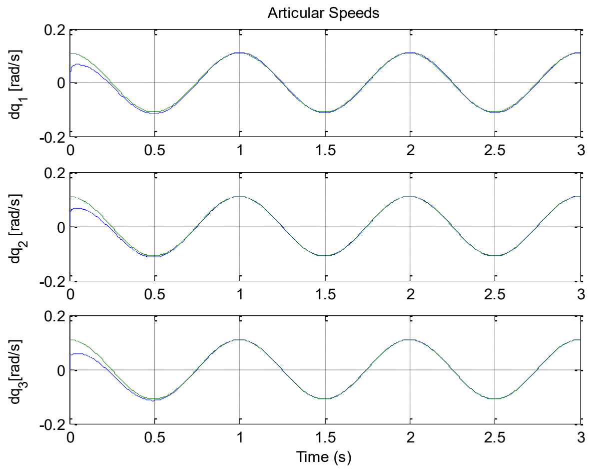

Figure 7. Comparison of articular reference speed trajectory and FPID controlled trajectory.

In comparing Figure 6 (PID approach) and Figure 7 (FPID approach) for the articular speed reference trajectory:

- ***Amplitude Differences***: The FPID controller (Figure 7) appears to have slightly smoother and less fluctuating curves than the traditional PID controller (Figure 6). This can indicate that the FPID controller reduces overshoot or produces a more damped response compared to the PID.
- ***Response Consistency Across Joints***: In Figure 6, the three joints exhibit similar but slightly varying amplitudes and responses. Figure 7, however, shows more uniformity in amplitude across all joints, suggesting that the FPID may achieve better coordination among the joints.
- ***Initial Response***: At the beginning of the time range, Figure 7's initial response shows a softer, less aggressive curve compared to Figure 6. This could imply that the FPID controller provides a more gradual acceleration, potentially resulting in less stress on the robotic joints.
- ***Phase Difference***: There is a slight phase shift in the initial cycles between the PID (Figure 6) and FPID (Figure 7) responses, which might indicate that the FPID has a different dynamic response time.

In summary, these differences suggest that the FPID controller may provide smoother, more coordinated, and possibly less aggressive control over the robotic joints, which might be beneficial in applications requiring precise and gentle movement. Let me know if you'd like further analysis or additional details.

Figure 8 illustrates the torque of the robot utilizing the PID controller, whereas Figure 9 depicts the torque of the robot employing the FPID controller.

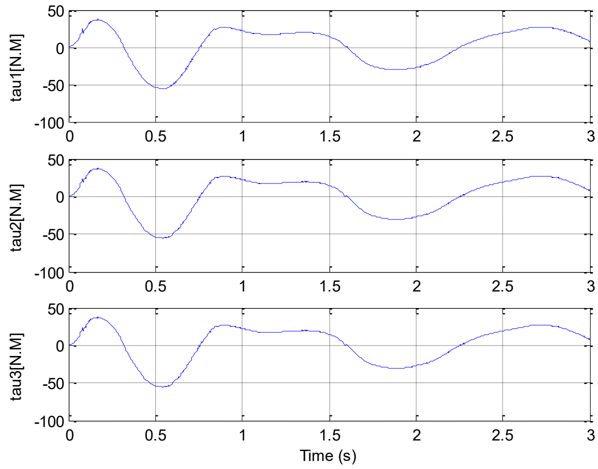

Figure 8. Torques of the PID controller.

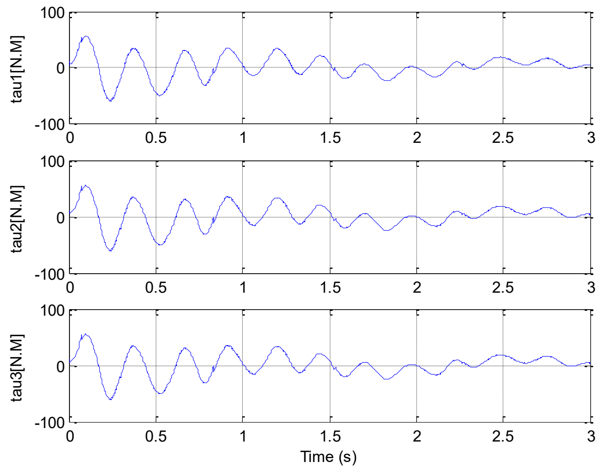

Figure 9. Torques for the FPID controller ($\alpha = 0.5$).

In comparing Figure 8 (PID approach) and Figure 9 (FPID approach) for the articular speed reference trajectory:

- ***Response Smoothness***: The PID-controlled system (Figure. 8) shows smoother waveforms with larger oscillations and slower settling times. This suggests that the PID controller may provide adequate control but allows some degree of overshoot or oscillation, which takes time to damp out.
- ***Response Oscillations***: The FPID-controlled system (Figure. 9) shows a more oscillatory behavior at a higher frequency but with reduced amplitude. This typically indicates that the FPID controller is introducing higher-frequency adjustments, which may help reduce steady-state error and improve precision.
- ***Amplitude and Stability***: In Figure 9, the oscillations in the FPID control response appear to dampen more quickly than in Figure 8, implying that the FPID controller might be more effective in quickly reducing error. The finer oscillations may mean the FPID is adjusting the control action more frequently, a characteristic often beneficial for precision tasks in robotics.
- ***Settling Time***: Overall, the FPID response suggests potentially faster stabilization (settling time) when compared to PID, a key advantage of fractional-order controllers in fine-tuning dynamic systems.

In summary, Figure 8 (PID) control results in smoother but slower stabilization, while Figure 9 (FPID) control provides quicker damping of oscillations, potentially leading to a faster and more precise response. This makes FPID potentially advantageous for applications in SCARA robots where high precision and fast response are critical.

Table 2 provides a performance comparison between the proposed FPID controller and the classical APID controller for controlling a SCARA robot. Table 2 summarizes the transient response stability parameters, including overshoot (OS[%]), settling time (T_S[s]), and delay time (T_D[s]), for different configurations of the FPID with varying values of the fractional parameter α. As can be seen from the table:

- ***Overshoot****:* The FPID controller shows a range of overshoot values based on α\alphaα. Generally, the FPID with $\alpha = 0.3$ has a slightly higher overshoot (0.23) than the baseline FPID (0.21), but it remains lower than the APID's overshoot of 0.311. The overshoot varies slightly as α changes, with values ranging from 0.21 to 0.27.
- ***Settling ime****:* The FPID controller has a faster settling time than the APID. The FPID settles in around 0.27 seconds across varied α values, while the APID takes longer at 0.48 seconds.
- ***Delay Time****:* The FPID controller also features shorter delay lengths, ranging from 0.0011 to 0.0015 seconds, compared to the APID's delay time of 0.004 seconds. This implies enhanced reactivity of the FPID.

Table 2. Transient response stability parameters

Controller	**OS[%]**	$\boldsymbol{T_s}$**[s]**	$\boldsymbol{T_D}$**[s]**
APID	0.311	0.48	0.004
FPID	0.21	0.272	0.0012
FPID ($\alpha = 0.3$)	0.23	0.273	0.0011
FPID ($\alpha = 0.5$)	0.22	0.271	0.0013
FPID ($\alpha = 0.7$)	0.27	0.277	0.0015
FPID ($\alpha = 0.9$)	0.26	0.278	0.0014

Overall, the proposed FPID controller enhances transient performance by reducing overshoot, settling time, and delay time, and these benefits are sustained throughout a wide range of fractional orders. This makes it a viable alternative to conventional APIDs for better SCARA robot control.

6. Conclusion

This study decreases temporal lag while tracking variable trajectories using a typical PID controller by including a fractional integrator into the PID design. This update aimed to reduce latency and enhance asymptotic tracking by using the improved dynamic features of fractional-order systems, as established in previous studies. Simulation findings reveal that the proposed fractional-order PID (FPID) controller outperforms the traditional APID controller in terms of trajectory tracking accuracy, tracking error reduction, stability enhancement, and oscillation and overshoot avoidance. Unlike the APID, which has amplitude and phase variations, the FPID provides a more consistent and reliable response.

Additionally, the proposed FPID controller outperforms the PID controller in maintaining constant amplitude, particularly at cycle peaks and troughs, which leads to better amplitude regulation and phase alignment. It produces smoother, less variable speed reference trajectories that exhibit improved damping and less overshoot. The proposed FPID improves uniformity and synchronization across joints, which is critical for correct robotic motions, while its gentler initial reaction reduces joint stress, extending system life.

Furthermore, the proposed FPID controller has a different dynamic reaction time than the PID controller, as shown by the phase shift, which is beneficial in circumstances requiring precise phase alignment. It increases tracking accuracy, stability, and consistency while lowering component stress. This approach may be used to a variety of fractional and integer-order systems to increase performance and noise rejection, making it especially helpful in complex control applications such as robotics, where precision and reliability are crucial.

Conflicts Interest Statement

The authors declare that they have no known conflicts financial interests or personal relationships that could have influenced the work reported in this article.

Data Availability Statement

No data or additional materials were utilized for the research described in the article.

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