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Development of an Adaptive Multi-Criteria Optimization Framework Using the PSI Method: Validation via Thermal Insulation Selection

Srđan Vasković[1*], Petar Gvero[2], Milovan Kotur[2], Đorđe Vojinović[3], Ljubiša Preradović[4], Danijela Kardaš-Ančić[2], Milan Pupčević[2]

[1]*Faculty of Mechanical Engineering, University of East Sarajevo, 71123 East Sarajevo, Bosnia and Herzegovina.*
[2]*Faculty of Mechanical Engineering, University of Banja Luka, 78000 Banja Luka, Bosnia and Herzegovina.*
[3]*Faculty of Technology, University of Banja Luka, 78000 Banja Luka, Bosnia and Herzegovina.*
[4]*Faculty of Architecture, Civil Engineering and Geodesy, University of Banja Luka, 78000 Banja Luka, Bosnia and Herzegovina.*

Abstract

Multi-criteria optimization is primarily utilized to rank various alternatives and project types. However, the question arises: is it possible to use the multi-criteria optimization process for something more than that, in terms of determining the actual set of optimal criteria and creating optimal paths of their changes according to desired constraints? In this regard, it is necessary to find a connection between the output provided by multi-criteria optimization method itself and a hypothetical alternative that represents the current optimal alternative, referred to as a synthetic profile or transit optimum, which does not represent any initial alternative in the optimization matrix. Such an actual transit alternative is formed by the mutual influence of all factors in the decision matrix, but also depends on the multi-criteria optimization process itself. This requires the creation of a mathematical feedback loop between the optimization process and the redistribution of all alternatives according to the criteria, to create a new transit optimal set of criteria. As a representative method for multi-criteria optimization, the PSI (Preference Selection Index) method was selected because of its advantages in terms of its integrated procedure for calculating criteria weights and its objectivity in categorization. The proposed adaptive PSI framework introduces a synthetic profile that enables iterative feedback and two-stage optimization, transforming the classical static ranking into a dynamic adaptive process with a clear methodological advancement over existing MCDM techniques. The overall concept was tested on the optimization of thermal insulation thickness for walls under the conditions of Bosnia and Herzegovina.

Keywords: PSI method, Adaptive multi-criteria optimization, Synthetic profile, Feedback loop optimization, Two-stage optimization, Optimal insulation thickness

1. Introduction

The application of multi-criteria optimization has gained significant intensity in the last 30 years [1]. The development of both new methods and their applications in various problems is an obvious indicator. At the same time, the speed of human decision-making has begun to unconsciously compete with machines, while the subjective feeling of the human self has changed substantially. In this context, there is a growing need for mathematically simpler and more objective multi-criteria decision-making (MCDM) methods. The Preference Selection Index (PSI) method, developed by Maniya and Bhatt [2], is an objective MCDM technique that eliminates subjective weight assignment by determining criterion weights through statistical evaluation of normalized data. Its advantages include mathematical

Corresponding author: Srđan Vasković (e-mail: srdjan.vaskovic@ues.rs.ba)
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simplicity, transparency, and applicability to both quantitative and qualitative criteria [3]-[10]. However, like other static MCDM methods, classical PSI provides only a one-time ranking without inherent mechanisms for iterative improvement or feedback. Main limitations of PSI method relate to: lack of attribute weighting [11], simplistic and inflexible approach [12], inadequacy in complex or detailed analyses [13], inadequate integration with the other decision-making techniques [14], oversimplification in multi-criteria decisions [15], real-world applications challenges [16]. The PSI method is also used in hybrid models where it is used as a reliable mechanism for automatically determining the weights of criteria that are then used in other ranking methods [17], [18].

The term "Adaptive Multi-Criteria Optimization" represents an advanced part of mathematical optimization that simultaneously solves two or more conflicting objectives, while the optimization system adjusts its parameters or model during the solution process. Simply defined, adaptive multi-criteria optimization enables "smarter" decision-making by not only offering a static answer, but also evolving along with the problem it solves. This practically means that it does not only provide a ranking of variant solutions, but is used in systems of multiple coupled equations and conditions that describe a specific problem [19], [20]. Adaptive optimization systems do not consider the problem as a static structure, but use feedback to adjust the model. In recent years, this approach has become a standard in the integration of machine learning and MCDM [21], [22].

The MARCOS (Measurement of Alternatives and Ranking according to COmpromise Solution) [23] and ARIE (Adaptive Ranking with Ideal Evaluation) [24] methods share the same basis in terms of: adaptability to ideal solutions and stability of the obtained ranked solutions. Thus, they represent a link to adaptive MCDM.

The main task of this study is the development of an adaptive PSI framework that extends the classical method through the introduction of a synthetic profile (transit optimum) – an imaginary alternative generated according to the PSI preference indices and the initial values at the optimization matrix in one iteration performed. This way enables a feedback-driven iterative process and two-stage optimization, transforming static ranking into adaptive multi-criteria optimization. In addition, it is necessary to mathematically ensure that all elements of this process are consistent and functional, such as the application of normalization of criterion sizes, the numerical values used (especially zero), and the dispersion of the system of analyzed values itself. This work differs from previous adaptive MCDM approaches [19]-[24] in that it derives the feedback mechanism directly and objectively from the PSI preference structure itself, without requiring external weighting or machine-learning modules. More specifically, the novelty of this study is that the PSI method is not used only as a static ranking technique, but is extended into an adaptive optimization framework. The PSI preference indices are transformed into participation coefficients which are then used to construct a synthetic profile, which representing a PSI-derived projection of the initial decision space. Within the the proposed framework, the synthetic profile acts as an adaptive optimization operator by transferring information contained in the PSI ranking back into the decision matrix as a transitional optimization state. In this way, it enables feedback-based iteration, convergence analysis, and two-stage optimization coupling.

Although this paper analyzed optimal thickness insulation from expanded polystyrene (EPS), recent studies have shown that bio-based insulation materials, such as coconut shell and kenaf fiber composites, can also achieve thermal conductivity values of around 0.041 W/mK, which are very similar to those of EPS. This indicates that the wider adoption of bio-based insulation materials can be expected in the future [25, 26]. Considering that the choice of thermal insulation thickness is a direct techno-economic issue, it indirectly and practically affects influences overal building performance, energy consumption, heating or cooling source power, thermal imbalance, moisture problems, comfort in residential buildings and much more [27]. The proposed adaptive multi-criteria optimization framework is validated through the selection of optimal thermal insulation thickness for detached single-family houses in Bosnia and Herzegovina, which according to Typology of Residential Buildings in Bosnia and Herzegovina that represents over 70% of the national residential stock [28].

2. Methodology

In this paper PSI method is used as the computational basis for developing an extended PSI-based MCDM framework. The proposed extension introduces a synthetic profile generated from PSI-derived percentage contributions of the alternatives. Upgrading the PSI method in terms of introducing the distribution of the preference index coefficient across the appropriate alternatives and generating a transitional optimization state based on the criteria values and PSI-derived preference structure generated from the initial optimization matrix and the optimization process. This represents an initial step toward the development of an adaptive PSI-based MCDM framework. Basically, the entire PSI methodological concept developed by Maniya and Bhatt [2] is used in this paper to extend and connect the preference selection index with the initial optimization matrix.

The paper proposes a methodological way of obtaining a vector formed by percentages calculated using the preference selection index. Each alternative from the initial matrix has its own percentage value in forming the final solution, which we will call the so-called "synthetic profile". The synthetic profile is mathematically equivalent to a convex (barycentric) combination of alternatives, but it differs conceptually because the weighting coefficients are derived from the concept of the PSI preference structure and the resulting profile is used as a transitional optimization state within an adaptive decision process. The novelty of the proposed approach does not lie in the mathematical form of the synthetic profile, which corresponds to a convex combination, but in its derivation from PSI preference indices and its use as a feedback-based transitional state within the decision-making process. Later in the paper, it will be shown that the synthetic profile is actually an imaginary vector made up of optimization criteria that is created as the product of the vector of percentages formed by the preference selection index and the initial optimization matrix [29], [30]. This synthetic profile retains the information about the performed optimization process, but also about all the real influences of the alternatives and their criteria from the initial optimization matrix. In this way, a feedback loop can be created between the created synthetic profile and the multi-criteria optimization process itself. This paper proposes only the static MCDM process and the process of eliminating the lowest-ranked alternatives in the simple iterative process. Also, the possibility of applying two-stage optimization based on the use of a synthetic profile will be demonstrated. The synthetic profile is used to couple the first and second stages of optimization.

The datasets related to thermal insulation thickness were taken from the Typology of Residential Buildings in Bosnia and Herzegovina [28] and facade cost data for Bosnia and Herzegovina [31], and were used to validate the proposed adaptive PSI framework. Accordingly, the methodology is organized into four connected stages: presentation of the classical PSI method, derivation of the synthetic profile, implementation of the static adaptive PSI procedure, and demonstration of two-stage optimization through the thermal insulation case study.

3. Description and Classic Applications of the PSI Method

PSI method in multi-criteria optimization provides results by using minimal and simple calculations. This method was developed by Maniya and Bhatt [2] and belongs to the group of objective multicriteria decision making methods. The PSI method in multi-criteria decision-making (MCDM) is based on a statistical concept. The logic of this method uses the statistical concept of sample variance to determine the variation of preferences for each criterion. In terms of determination of criterion weights, this process is integrated within the PSI method. In this method can be used for any number of attributes. It can also be used for both quantitative, or calculation criteria and qualitative or descriptive criteria [2], [4], [32]. Qualitative criteria must be transformed into a numerical format before applying the PSI method. The following are the steps and formulas related to the PSI calculation and methodology [2].

Step: 1. Define the problem: Determine the objective and identification important attributes and alternatives involved in the decision-making problem.

Step: 2. Formulation of the decision matrix: This step involves design of a matrix based on optimization task related to information available that describes the criteria of the problem. In decision matrix each row is allocated to one alternative, and each column to one attribute or criteria. Therefore, an element X_{ij} of the decision matrix X gives the value of the i-th alternative and j-th criteria in original real values. In creating a decision matrix, m is the number of alternatives and n is the number of criteria.

$$X_{ij} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}$$

Step: 3. Data normalization: In the MCDM methods it is required to make the criteria value dimensionless. For this purpose, the criteria values are transformed in interval from 0 to 1. This process of transforming criteria is normalization which depends on the type of criteria. The accuracy of further calculations in optimization depends a lot on the normalization process. In the basis of the PSI method, the most significant is the ordinary linear normalization, which was proposed by the authors of this method [2]. For beneficial type of criteria, larger values are better and can be normalized as Eq. (1, 2):

$$N_{ij} = \frac{X_{ij}}{\max(X_j)} \tag{1}$$

For non-beneficial type, then smaller values are desired, and can be normalized as:

$$N_{ij} = \frac{\min(X_j)}{X_{ij}} \tag{2}$$

Where Xij is the criteria index (i =1, 2, ..., m and j =1, 2, ..., n).

In this paper, special attention is paid to the process of applying the criteria normalization, according to previous research from [33]. In this regard, the methods: linear normalization, max linear normalization, Jüttler-Körth normalization, and Z-score normalization are suitable methods to combine with the PSI method. All of the above normalization methods can be used when there are no zero values in the initial matrix. When there exists a certain value Xij = 0, then the linear method cannot be applied. The remaining three methods (max linear normalization, Jüttler-Körth normalization, and Z-score normalization) can still be applied. In cases when there exists a value max (Xij) = 0, then all three methods (linear normalization, max linear normalization, Jüttler-Körth normalization) cannot be applied. In this case, Z-score normalization is used. This systematization of normalization methods has a very good contribution in terms of greater use of the PSI method [33].

Step: 4. Calculation of mean values of normalized data, Eq. (3):

$$\overline{N_j} = \frac{1}{m}\sum_{i=1}^{m} N_{ij} \tag{3}$$

Step: 5. Determination the preference values from the mean values by Eq. (4):

$$\phi_j = \sum_{i=1}^{m}\left[N_{ij} - \overline{N_j}\right]^2 \tag{4}$$

Step: 6. Determination the deviation in the preference values by Eq. (5):

$$\theta_j = \left[1 - \phi_j\right] \tag{5}$$

Step: 7. Calculation of overall preference value (weights) for the criteria by Eq. (6):

$$\beta_j = \frac{\theta_j}{\sum_{j=1}^{n}\theta_j} \tag{6}$$

Step: 8. Calculation the PSI_j index of each alternative by Eq. (7):

$$PSI_j = \sum_{i=1}^{n} N_{ij} \cdot \beta_j \tag{7}$$

The highest PSI value corresponds to the best ranked alternative. In this regard, the PSI value will be used as an auxiliary value for calculating the synthetic profile. The following consideration will discuss about the possibilities of mathematically creating a synthetic profile. This approach represents an introduction to framework of adaptive MCDM. However, what if we could establish a self-adjusting optimization process that evolves toward the desired optimum? In this regard, there is also the main motive for moving from the classical application of ranking alternatives to the domain of adaptive optimization using the PSI method. All this opens up some new possibilities for the application of the PSI method and MCDM optimization in general.

4. Creation of a Synthetic Profile and Transit Optimum Based on the PSI Method

The process of defining the synthetic profile is methodologically completely linked to the general concept of PSI and the obtained values of the preference index PSI_j according to the considered alternatives. The percentage value of the participation of individual variants in the creation of the synthetic profile can be obtained on the basis of the percentages calculated from the individual value of the preference index PSI_j

corresponding to a certain alternative and the total sum of all preference index, depending on the number of considered alternatives, Eq. (8):

$$p_j = \frac{PSI_j}{\sum_{j=1}^{m} PSI_j} \tag{8}$$

Basically, in the previous way, a vector of percentages p_{i1} is formed, of which each alternative has its own percentage of influence, i.e. i=1...j.

$$P_i = \begin{bmatrix} p_1 & p_2 & \dots & p_j \end{bmatrix}$$

The synthetic profile Si Eq. (9), is actually an imaginary alternative, which is created as a vector product of the vector of percentages formed by the preference selection index Pi and the initial optimization matrix Xij [29], [30]. Through this coupling process, in addition to the fact that the synthetic profile is mathematically coupled by the product of the vectors Pi and Xij, it is completely dependent on the optimization process through the PSI method.

$$S_i = P_i \times X_{ij} = \begin{bmatrix} s_1 & s_2 & \dots & s_n \end{bmatrix} \tag{9}$$

The coefficients p_i represent the normalized contribution of each alternative to the synthetic profile. Since they are obtained by dividing each PSI_i value by the total sum of all PSI_i values, they satisfy:

$$p_i \geq 0, \qquad \sum_{i=1}^{m} p_i = 1,$$

provided that all PSI values are non-negative. In this formulation, $i = 1, \dots, m$ denotes the alternatives, while $j = 1, \dots, n$ denotes the criteria. The synthetic profile is therefore obtained as the vector-matrix product, $S = PX$, where $P = [p_1, p_2, \dots, p_m]$ is the PSI-derived participation vector and $X = [x_{ij}]$ is the initial decision matrix. Each component of the resulting vector S corresponds to one criterion and can be written as:

$$S_j = \sum_{i=1}^{m} p_i x_{ij}, \qquad j = 1, \dots, n.$$

In this way, the synthetic profile is calculated in the original decision space and preserves the physical meaning and units of the criteria. The feedback mechanism is created by replacing the worst-ranked alternative in the current decision matrix with the newly obtained synthetic profile and then repeating the PSI procedure until the ranking becomes stable or the profile converges toward the selected optimum. In this paper, the term synthetic profile denotes the generated decision vector, while transit optimum refers to its role as an intermediate state in the adaptive optimization process.

As an example for calculating the value of the synthetic profile, the values related to the thickness of the thermal insulation of the outer shell of a free-standing house, built in the interval from 1981 to 1991, were taken from Typology of Residential Buildings in Bosnia and Herzegovina [28]. By distribution of typologies of residential buildings in BiH by gross surface, single-family houses take about over 70%. Single-family house with ground floor and attic used for residential purposes. The roof is traditional, wooden, gable roof, with clay roof tiles. External walls are made of 29 cm hollow clay bricks, with a 5 cm thermal insulation layer in the contact facade system. Three measures were taken into consideration for wall insulation: improvement 1 (thickness of thermal insulation of 10 cm), improvement 2 (thickness of thermal insulation of 20 cm) and improvement 3 (passive house, thickness of thermal insulation of 30 cm). Thermal insulation characteristics correspond to the value of λ = 0.041 W/mK, i.e. expanded polystyrene (EPS) - white polystyrene. Certain categories of white polystyrene (such as EPS 50 or EPS 70) have thermal conductivity in this range (λ = 0.039–0.041 W/mK). Table 1 provides an overview of the alternatives that represent improvement measures 1, 2 and 3, as well as criteria for savings in heat transmission loss through the wall, and investment in performing 1 m^2 of wall insulation in all three improvements. Due to the simplicity of the calculation that we want to show regarding the synthetic profile, only two criteria were taken into consideration. The criterion of savings in heat loss is a benefit criterion (+), while the investment per 1 m^2 of insulation is a non-benefit criterion (-). Investment in performing 1 m^2 of wall insulation corresponds to prices in Bosnia and Herzegovina [31].

Table 1. Overview of the alternatives that represent improvement measures and criteria.

Alternatives/criteria	Savings in heat loss through the wall (W/K)	Investment in performing 1 m^2 of wall insulation (BAM/m^2)
Improvement 1	48	50
Improvement 2	62	70
Passive house	78	110

By applying the classic PSI method and usual linear normalization of the initial data in Table 1, the PSI index values was calculated by (Eq. (1) to Eq. (7)). As well as the ranking of the alternatives in the following Table 2 gives an overview.

Table 2. Overview of the ranking of improvement measures and PSI index by using linear normalization data.

Alternatives/criteria	PSI index	Ranking
Improvement 1	0.799	1
Improvement 2	0.756	2
Passive house	0.738	3

From Table 2 it is clearly seen that the optimal value is the alternative on the first place, which refers to Improvement 1, which is thermal insulation of 10 cm thickness of wall. This alternative also corresponds to the highest value of the PSI index and, respectively, the rank number corresponds to the value of the PSI index. At the same time, this scale confirms and demonstrates the usability of the PSI method in ranking building renovation measures in both optimal and hierarchical order. Much in all MCDM processes depends on the process of normalizing numerical values from the initial matrix, which will be discussed later. Using standard classical formulations for PSI (Eq. (1) to Eq. (7)) but also non-standard proposals via equations (Eq. (8) and Eq. (9)), the value of the synthetic profile or imaginary alternative S_{p1} can be calculated, Table 3.

Table 3. The value of the synthetic profile or imaginary alternative by using linear normalization data.

Alternatives/criteria	Savings in heat loss through the wall (W/K)	Investment in performing 1 m^2 of wall insulation (BAM/m^2)
Synthetic profile S_{p1}	62.272	75.908

Synthetic profile (imaginary alternative), in practical terms, represents the reduction of initial information from the starting matrix through an optimization process to a new value that can be used to expand the MCDM process for new purposes. Table 3 shows that the synthetic profile is positioned between the initial insulation alternatives, because it contains the weighted influence of all alternatives according to their PSI-derived participation coefficients. This confirms that the synthetic profile is not a separately selected real alternative, but a generated transitional state that summarizes the information contained in the initial decision matrix.

4.1. Adaptive static PSI optimization

The adaptive PSI procedure extends the classical PSI method by introducing an iterative feedback step. After the classical PSI index is calculated and the alternatives are ranked, the PSI values are transformed into participation coefficients and used to generate a synthetic profile. The worst-ranked alternative is then replaced by this profile, and the PSI procedure is repeated. Therefore, the adaptive implementation consists of the following sequence: defining the initial decision matrix, selecting the normalization method, calculating PSI indices, ranking alternatives, generating the participation vector, calculating the synthetic profile, replacing the worst-ranked alternative, and repeating the procedure until the ranking becomes stable or the profile converges toward the selected optimum. In this way, the system is naturally "pushed" towards the optimal solution. For this purpose, a series of transit optima are created that converge towards the desired optimal state. Figure 1. shows the general scheme of adaptive PSI, by using a previously created synthetic profile.

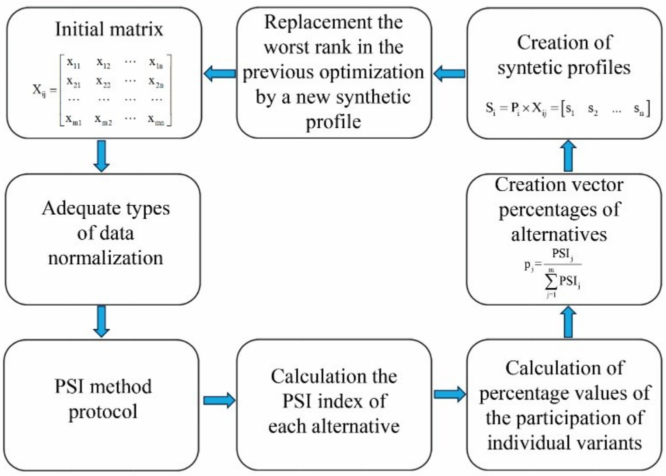

Figure 1. Basic concept of adaptive PSI optimization framework.

Table 4 shows the calculated values of 6 synthetic profiles. These values were generated in an iterative process where the main goal was to eliminate all the worst ranks in the initial matrix to the initially natural convergence by using the system of equations from (Eq.

(1) to Eq. (9)), as well as the basic concept of adaptive PSI optimization framework in Figure 1. In our case natural convergence is Improvement 1. So, the first synthetic profile convergence to the optimal solution shown in Table 4, green color row marked. The amounts to savings in heat loss: 52.672 W/K, and for investment 56.759 BAM/m^2. Linearly observing the scale of savings in heat loss, this savings corresponds to an insulation thickness of approximately 13 cm. The same conclusion can be reached by observing the scale for investments, because both scales are linked by the methodology itself and the synthetic profile. The values in Table 4 demonstrate the iterative behavior of the adaptive PSI procedure under linear normalization. Each newly generated synthetic profile moves the decision space closer to the selected optimum, while the worst-ranked alternative is gradually neutralized. Therefore, Table 4 does not only present numerical results, but also illustrates the convergence mechanism of the proposed adaptive PSI framework.

Table 4. Overview of synthetic profiles in optimal transit according to the selected optimum by using PSI and linear normalization.

Alternatives/criteria	Savings in heat loss through the wall (W/K)	Investment per 1 m^2 of wall insulation (BAM/m^2)
Synthetic profile S_{p1}	62.272	75.908
Synthetic profile S_{p2}	57.281	65.044
Synthetic profile S_{p3}	55.671	61.543
Synthetic profile S_{p4}	55.146	60.401
Synthetic profile S_{p5}	54.973	60.025
Synthetic profile S_{p6}	52.672	56.759
Optimal alternative by classic PSI and linear normalization	48	50

Similarly, using max linear normalization [33] and PSI method, we obtain the distribution of synthetic profiles in Table 5. Max linear normalization uses formulas for normalizing data in the initial matrix according to the benefit or non-benefit criteria Eq. (10, 11). It is especially worth noting that we use this type of normalization when we do not have maximum zero values within one criterion observed.

$$N_{ij}=\frac{X_{ij}}{\max(X_j)} \tag{10}$$

$$N_{ij}=1-\frac{X_{ij}}{\max(X_j)} \tag{11}$$

Table 5. Overview of synthetic profiles in optimal transit according to the natural convergence solution by using PSI and max linear normalization.

Alternatives/criteria	Savings in heat loss through the wall (W/K)	Investment in performing 1 m^2 of wall insulation (BAM/m^2)
Synthetic profile S_{p1}	62.126	75.470
Synthetic profile S_{p2}	57.179	64.782
Synthetic profile S_{p3}	55.585	61.381
Synthetic profile S_{p4}	55.067	60.277
Synthetic profile S_{p5}	52.815	57.116
Optimal alternative according to adaptive PSI and max linear normalization	48	50
Optimal alternative according to classic PSI and max linear normalization	62	70

Looking at the synthetic profile calculations in Table 4, it can be concluded that the neutralization of the worst ranks in the first case when using adaptive PSI framework and ordinary linear normalization converges to the optimal variant after the sixth iteration. In the case of the adaptive PSI method framework and max linear normalization, the neutralization of the worst ranks occurs after five iterations, Table 5. However, it is important to note that when using max linear normalization and PSI, the optimal variant in the first optimization was improvement 2. After calculating the synthetic profiles and transit optima, in both cases it was concluded that both cases converge to the improvement measure 1. The final synthetic profiles Sp_6 and Sp_5 show good agreement. Table 5 is particularly important because it shows that max-linear normalization initially produces a different classical PSI optimum, while the adaptive PSI procedure still converges toward Improvement 1. This indicates that the proposed feedback mechanism can be used to test the stability of rankings obtained under different normalization procedures.

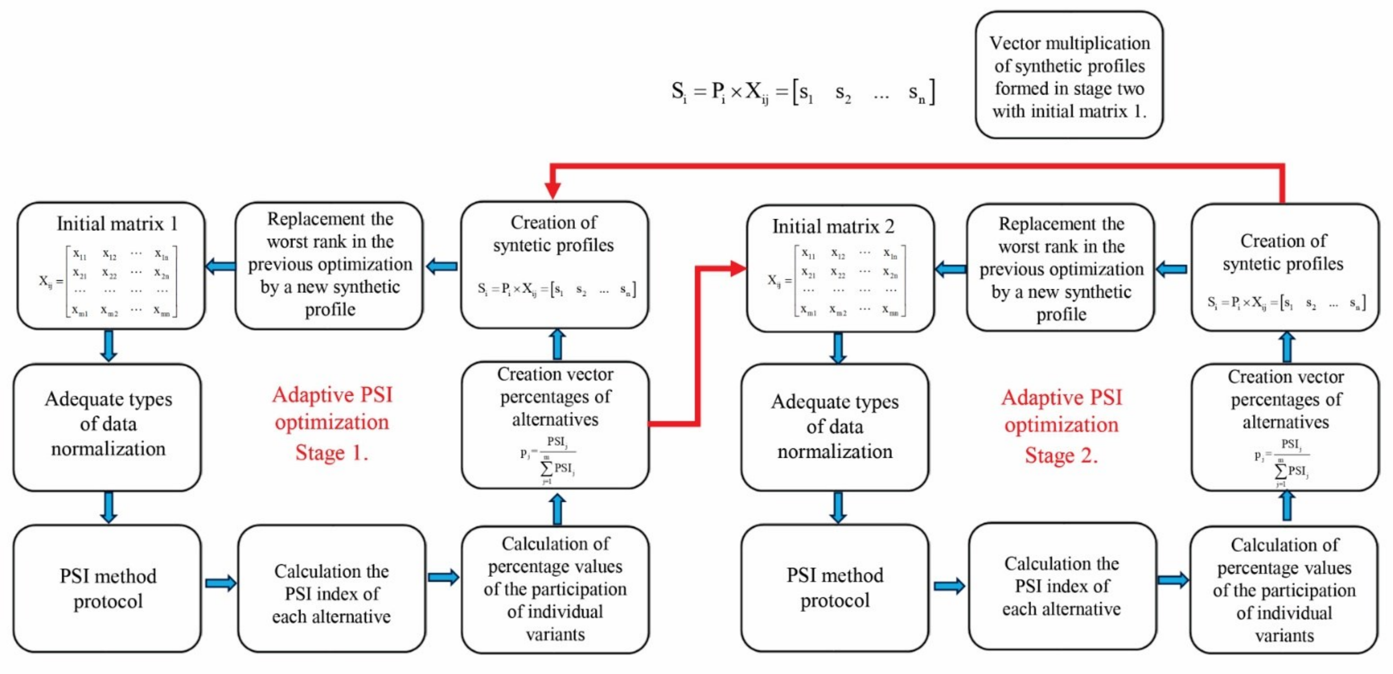

Figure 2. Possibilities of coupling first and second stage MCDM optimization by PSI method.

5. Possibilities of Coupling First and Second Stage MCDM Optimization by Using Synthetic Profile

There are processes in multi-criteria optimization that should be coupled and executed in two or more stages. This is justified in several aspects: avoiding the influence of one optimization process on another in terms of the influence of "mathematical noise", the possibility of controlling by optimization stages, the hierarchy of variability of optimization parameters in different stages, etc. Basically, the key to linking different stages of MCDM optimization is actually mutual dependence on the influence vector percentage of alternatives and the vector of synthetic profiles (Eq. (8) and Eq. (9)), or their mutual mathematical coupling. The following Figure 2. shows a scheme of the possibility of coupling two stages of MCDM by using the PSI method.

In the literature, approaches such as two-stage optimization, bilevel optimization and iterative MCDM frameworks have been investigated. However, the combination of PSI-based preference modeling, synthetic profile generation and adaptive second-stage optimization appears to be relatively rare [34], [35].

Basically, the entire proposed concept of two-phase optimization using the PSI method is based on the previously proposed adaptive PSI framework. The main point lies in the logical and algorithmic pairing of the first and second optimization stages. This pairing consists of the following list of assumptions in next.

1. The output from the first stage of optimization in the form of calculated impact of percentages of considered alternatives from the initial matrix 1, is used as an input to the initial matrix 2 in the second stage.

2. In the second stage of optimization, all alternatives from the first stage are used as criteria. The reason for this is simple, so that we can control of them in the optimization process.

3. Alternatives in the second stage consist of: calculated percentages of considered alternatives from stage 1, percentage values of the initial state and percentage values of the final state that is to be achieved in terms of strategic planning or control of the second optimization segment.

4. The sign of the criterion in the second stage (benefit or non-benefit) is determined based on the difference between the state we want to reach and the initial state. The percentage of influence of the considered alternatives from the first stage is neutral and does not affect the sign of optimization.

5. After the calculation and execution of the optimization process in stage 2. by the PSI method, a new synthetic profile of the considered percentages is created.

6. Subsequently, the values of the synthetic profiles formed in phase two of the optimization are vector-multiplied by initial matrix 1, creating double-optimal values. The impact percentages of the alternatives considered in the first optimization phase remain constant during the second phase. The desired alternative in matrix 2 is managed by varying the achieved objective values within an interval of 0 to 1 (i.e., 0% to 100%). This process facilitates the development of a group of scenarios that represent an optimal transit toward the convergent optimum from matrix 1. The primary prerequisite is the fulfillment of the convergence criteria for both optimization stages 1 and 2, which must be rigorously examined beforehand.

7. It is safe to say that the mutual connection between the two stages of optimization by PSI methods works on the approximate reciprocity between the stages. Stage one gives stage two an optimized percentage of alternatives, while stage two returns an optimized synthetic profile to stage one. Stage two takes into account its information structure through the optimization process, while stage 1 does the same again through the same defined synthetic profile and information structure from the initial matrix 1.

Below is a numerical example for two stage optimizations by PSI related to the degree of isolation of free-standing houses in Bosnia and Herzegovina, in accordance with the Typology of Residential Buildings in Bosnia and Herzegovina [28]. It is important to emphasize that each problem has its specific characteristics and requires a detailed approach to defining such models.

5.1. Numerical example of two-stage adaptive PSI optimization for calculation of transit optimum

The initial data for applying the PSI method in two phases to create a synthetic profile are very similar to the previous ones, except that they are expanded to the second phase in the form of percentage values relating to the current state of insulation of free-standing houses and the state we want to achieve in this regard. Table 6 presents the alternatives for the first optimization stage and contains the same criteria in the form of heat loss savings and investment. The only difference compared to Table 2 is that the current state is also considered as a new alternative with zero criteria values, i.e. without any actions.

Table 6. Overview of the alternatives that represent improvement measures and criteria for first stage, initial matrix 1.

Alternatives/criteria	Savings in heat loss through the wall (W/K)	Investment per 1 m^2 of wall insulation (BAM/m^2)	Ranking
Current state	0	0	3
Improvement 1	48	50	1
Improvement 2	62	70	2
Passive house	78	110	4

It should be emphasized that due to the occurrence of zeros in the first alternative, which refers to the existing state, the application of classical linear normalization cannot be applied. This leads to mathematical uncertainty. On the other hand, replacing zero values with finite small values of epsilon also leads to inaccuracy in the calculation. Therefore, it is advisable to apply Min-Max linear normalization. The rank in Table 6 was obtained by applying this normalization. Also, all further calculations in terms of adaptive two-stage PSI optimization were performed using this type of normalization. Min-Max linear normalization uses formulas for normalizing data in the initial matrix according to the benefit or non-benefit criteria Eq. (12, 13).

$$N_{ij} = \frac{X_{ij} - \min(X_j)}{\max(X_j)\text{-}\min(X_j)} \tag{12}$$

$$N_{ij} = \frac{\max(X_j) - X_{ij}}{\max(X_j)\text{-}\min(X_j)} \tag{13}$$

The creation of synthetic profiles up to the neutralization of the worst rank to the optimal alternative is shown in Table 7, for stage 1. Table 7 confirms that the first-stage adaptive PSI procedure generates a sequence of synthetic profiles directed toward the optimal insulation alternative. These profiles provide the techno-economic basis for the second stage, because the participation vector obtained from the first iteration is subsequently used as an input structure for scenario-based optimization.

Table 7. Checking convergence synthetic profiles towards optimum in stage 1.

Alternatives/criteria	Savings in heat loss through the wall (W/K)	Investment in performing 1 m^2 of wall insulation (BAM/m^2)
Synthetic profile S_{p1}	46.747	56.534
Synthetic profile S_{p2}	39.268	44.113
Synthetic profile S_{p3}	37.143	40.793
Synthetic profile S_{p4}	31.559	34.183
Optimal alternative by classic PSI and Min-Max linear normalization	48	50

The synthetic profile Sp_1 corresponds to the transit vector of percent alternatives from first iteration. With the percentage labels from p_1 to p_4 referring to: Current state (p_1), Improvement 1 (p_2), Improvement 2 (p_3), Passive house (p_4): p_1= 0.242651909; p_2= 0.269399531; p_3= 0.265252152; p_4= 0.222696408. These values are the input to the second stage of optimization. The initial matrix 2 for the second stage of PSI optimal transit calculation is shown in Table 8.

Table 8. Overview of the alternatives that represent improvement measures and criteria for second stage of PSI.

ALTS/ CRIT.	Current state (p_1)	Improv 1 (p_2)	Improv 2 (p_3)	Pass house (p_4)	Rank
Pi vector from st. 1	0.242	0.269	0.265	0.222	1
Current state	0.8	0.15	0.04	0.01	3
Desired state	0	0.8	0.15	0.05	2

In Table 8 regarding the alternative Current state and in accordance with the adopted criteria from p1 to p4 relating to the current state, improvement measure 1, improvement measure 2 and passive house, these values are assumed based on references [28], [31]. Over 80% of free-standing houses have poor insulation, while about 15% of houses have 10 cm insulation, about 4% of houses have 20 cm insulation, while passive houses are about 1%.

The desired state is moving towards to complete neutralization of poorly insulated free-standing houses, p1, increasing **improvement measure 1** to 80%, increasing **improvement measure 2** to 15% and **increasing passive houses** to 5%.

Regarding the determination of the sign of the criteria from p_1 to p_4 (benefit or no benefit criteria), as previously stated, the sign is determined based on the difference between the state we want to reach and the initial or current state. In this regard, p1 is non benefit (0-0.8) is negative. The other criteria, p_2, p_3 and p_4 have positive differentials and represent benefit criteria. The convergence conditions for synthetic profiles in stage 2 of optimization are satisfied, also. Synthetic percentage profiles Sp from stage two, calculated by adjusting values toward the desired state Ds for seven analyzed scenarios, are shown in Table 9.

Table 9. Synthetic percentage profiles from stage two optimization related to desired scenarios.

The desired state/ Synthetic profile	Curr. state (p_1)	Improv.1 (p_2)	Improv. 2 (p_3)	Passive house (p_4)
Ds_1	0.67	0.25	0.06	0.02
Sp_1	0.345	0.264	0.216	0.174
Ds_2	0.5	0.38	0.09	0.03
Sp_2	0.335	0.309	0.201	0.153
Ds_3	0.3	0.53	0.13	0.04
Sp_3	0.132	0.532	0.197	0.137
Ds_4	0.22	0.6	0.14	0.04
Sp_4	0.232	0.416	0.209	0.141
Ds_5	0.11	0.7	0.15	0.04
Sp_5	0.180	0.469	0.211	0.137
Ds_6	0.01	0.79	0.15	0.05
Sp_6	0.131	0.517	0.210	0.140
Ds_7	0	0.8	0.15	0.05
Sp_7	0.126	0.523	0.210	0.140

Table 9 shows how different desired-state scenarios affect the synthetic percentage profiles in the second optimization stage. The gradual reduction of the current-state share and the increase of improvement-related shares demonstrate how the proposed method can translate strategic renovation targets into optimized participation vectors. In this way, the table illustrates the role of the second stage as a scenario-control layer of the adaptive PSI framework.

By multiplying the vectors of the synthetic profiles Sp_n that determines the second stage from Table 9 with the initial matrix, Table 6 from the first stage, we obtain a "doubly" optimized synthetic profiles and optimal path to optimal **improvement measure 1** for thermal insulation of free-standing houses in Bosnia and Herzegovina. These profiles are dually optimized, adhering to the criteria from stage 1 and the scenario-based optimization from stage 2., the corresponding results are presented in Table 10.

Table 10. Optimized synthetic profiles by stage 1. and stage 2. optimization.

Double optimized synthetic profile	Savings in heat loss through the wall (W/K)	Investment in performing 1 m^2 of wall insulation (BAM/m^2)
Synthetic profile S_{p1}	39.69221958	47.52398381
Synthetic profile S_{p2}	39.30284273	44.11353534
Synthetic profile S_{p3}	42.41232287	49.45693436
Synthetic profile S_{p4}	44.01829462	51.0588161
Synthetic profile S_{p5}	46.40891789	53.44842874
Synthetic profile S_{p6}	48.83473207	56.0404912
Synthetic profile S_{p7}	49.06860188	56.27867102

Table 10 presents the final effect of coupling the first and second optimization stages. The obtained double-optimized synthetic profiles combine techno-economic criteria from the first stage with strategic scenario constraints from the second stage. The results show that the generated profiles approach the optimal insulation measure while remaining within a realistic investment range, which confirms the practical usefulness of the two-stage adaptive PSI procedure.

6. Results and Discussion

The previously discussed model of MCDM basically has techno-economic and strategic criteria. Of course, optimization processes depend on various other parameters, but the authors of this paper have focused almost exclusively on techno-economic and strategic research into transit optima related to the degree of insulation of free-standing houses. By applying different methods of calculating synthetic profiles as well as different measures of normalization of the initial values in the initial matrix, the following conclusions were reached in terms of two items. The first item refers to the application of the adaptive PSI method and the normalization methods for calculating synthetic profiles. The second item is related to optimal wall insulation thickness by using adaptive PSI and two stage optimizations.

In all analyzed cases, the optimal alternative determined by the PSI method was consistent with the results obtained using the VIKOR method with entropy-based weighting, indicating a high level of robustness of the PSI-based ranking [36].

6.1. Results and discussion about adaptive PSI method optimization

The application of adaptive optimization represents a challenge and a step beyond the classical MCDM application. In this regard, this paper analyzes the possibilities of such a concept using the PSI method and different types of normalization of the initial data. In this paper, three types of normalization of quantities were used: linear, max linear and min-max linear normalization. The biggest problem in the application of the PSI method is zero values, which can cause the problem of mathematical uncertainty, especially when classical linear normalization is applied. In the case that zero values are replaced with very small values of 10^{-6}, then there are an obvious disturbance and the appearance of inaccuracy in the range of observed alternatives. By applying max linear normalization, the problem of mathematical uncertainty can be avoided to some extent, but the stability of the optimal solution becomes very questionable. In this regard, by applying adaptive PSI and synthetic profiles, it is shown in the example given in Table 5. Additionally, the synthetic profiles converge toward very similar values when applying both linear and max-linear normalization (Sp_5 and Sp_6, Tables 4 and 5). The only difference lies in the convergence speed achieved with max-linear normalization, related to the number of iterations.

Figure 3 illustrates the progression of PSI index values for the generated synthetic profiles. The smooth evolution of these values indicates that the iterative replacement of the worst-ranked alternative does not produce unstable changes in the decision structure. Instead, the synthetic profiles form a gradual optimization path toward the selected optimum, which supports the interpretation of the synthetic profile as a transitional state in the adaptive PSI procedure. Figure 3 was created based on the data from Table 4. A smooth change in the PSI index values and a stable progression toward the optimal value Opt (Improvement measure 1) are clearly observable. In general, although the PSI values for the observed synthetic profiles evolve smoothly, the coupled criteria sometimes deviate from a slight increase or decrease in some cases. However, these anomalies are quickly corrected during the adaptive optimization process.

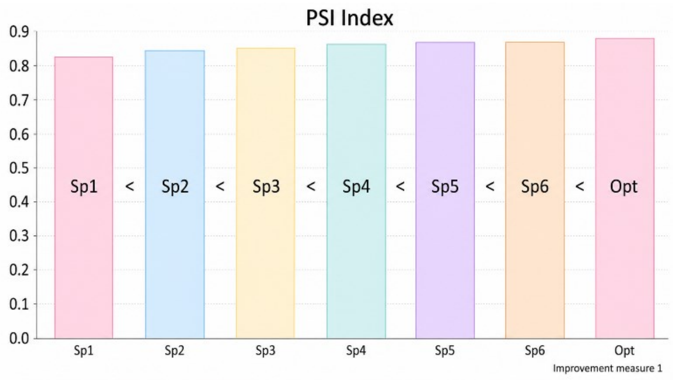

Figure 3. The stability of the synthetic profiles' progression toward the optimal value, optimized using the PSI method.

The key conditions for achieving stable convergence of the synthetic profile towards the optimum are the selection of an adequate type of normalization and the elimination of the worst ranks through the synthetic profiles to the final optimum. This process represents a natural progression towards an optimal state. This is still due to the static nature of the alternatives. It all happens without correcting their actual values, despite the changing synthetic profile used to eliminate the worst-ranked option.

In stability theory, Lyapunov functions are commonly used to examine whether the state of a dynamic or iterative system remains stable and progressively approaches an equilibrium or desired state [37]. In the proposed adaptive PSI framework, this idea is used to measure the distance between the generated synthetic profile and the stable optimal alternative. For n criteria, the Lyapunov-type potential can be defined as:

$$V^{(k)} = \sum_{j=1}^{n} wj \left(S_j^{(k)} - x_j^*\right)^2, \tag{14}$$

where $V^{(k)}$ is total squared distance of the synthetic profile from the optimum, wj is the weight coefficient of the criteria, $S_j^{(k)}$ is the normalized value of the j-th criterion in the synthetic profile at iteration k, n number of criteria and x_j^* is the corresponding normalized value of the stable optimal alternative. The adaptive PSI process is considered stable if this potential does not increase during the iterative replacement of the worst-ranked alternative: $V^{(k+1)} \leq V^{(k)}$. A sufficient condition for convergence is the existence of $\alpha > 0$ such that:

$$V^{(k+1)} - V^{(k)} \leq -\alpha V^{(k)}. \tag{15}$$

Under this condition, the Lyapunov-type potential decreases monotonically, which implies that the sequence of synthetic profiles converges toward the stable optimal alternative. Since the criteria are expressed in different units and numerical scales, the Lyapunov-type potential was calculated using normalized criterion values. For the basic adaptive PSI procedure in Table 4, the normalized Lyapunov potential decreases monotonically. However, the final double-optimized profiles in Table 10 show a clear stabilization around the techno-economic optimum. The minimum normalized potential is obtained for S_{p5}. This indicates that S_{p5} represents the closest compromise profile between techno-economic optimality and scenario-based strategic adjustment.

For the two-stage optimization, the PSI method was employed, using exclusively min-max normalization. The rationale behind this is to achieve convergent solutions from synthetic profiles toward the optimum. Notably, it should be mentioned that an entire alternative in initial matrix 1 contains zero values (see Table 6). It must be noted that linear min-max normalization is applicable only when the maximum criterion value in a given column is not equal to zero. Otherwise, alternative types of normalization must be utilized [33].

When the data dispersion term PV_j becomes large enough to produce negative transformed preference values through $\Phi_j = 1 - PV_j$, the dispersion terms can be rescaled before calculating Φ_j. Here, PV_j represents the preference variation, i.e., the dispersion of normalized values of the j-th criterion across all alternatives. The rescaled dispersion term is defined as:

$$PV_j^{norm} = \frac{PV_j}{\sum_{j=1}^{n} PV_j} \tag{16}$$

where PV_j^{norm} is the normalized preference variation and n is the number of criteria. Since:

$$0 \leq PV_j^{norm} \leq 1 \tag{17}$$

the transformed preference value remains non-negative, which prevents negative criterion weights. The proposed adaptive PSI framework integrates the synthetic profile, appropriate data normalization, Lyapunov-type convergence verification, and stabilization of the data-driven dispersion term into a useful optimization structure.

The critical question arises as to where to apply transient optima, synthetic profiles, and two-stage optimization in real-world scenarios. One of the potentials and highly significant applications is the calculation of the optimal energy mix under variable conditions and the creation of optimal scenarios aligned with a specific target [38]. The PSI method, interpreted in the previously described manner, demonstrates great potential for this purpose. Furthermore, the coupling of multiple distinct optimization spaces opens possibilities for extending this method toward the development of multi-phase optimization supported by PSI. Additionally, the basic framework of adaptive PSI can be utilized to enhance the accuracy of criteria weight calculations in hybrid approaches.

The practical implication of the proposed adaptive PSI framework is that it can support not only the selection of the best alternative, but also the definition of a transition path toward a desired state. In the insulation case study, the generated synthetic profiles represent intermediate renovation states that may be used for phased investment planning and scenario-based energy policy. The broader applicability of the method includes optimal energy mix planning, decarbonization pathways, HVAC system control, and other engineering systems with coupled techno-economic and strategic criteria.

6.2. Results and discussion about optimal wall insulation thickness by adaptive PSI method

As part of testing the core concept of the adaptive PSI method, a numerical dataset regarding the external wall insulation of detached houses in Bosnia and Herzegovina was utilized. A basic set of two criteria was employed: thermal loss savings (W/K) and the investment cost for 1 m2 of wall insulation (BAM/m^2). It can be stated with certainty that these two criteria are mutually independent and conflicting. Other criteria, such as financial savings and CO_2 emission reduction, are dependent on the thermal loss savings in this case. The results obtained through the adaptive PSI concept are presented in the following list.

1. The optimal wall insulation measure converges very closely to the value of 10 cm of insulation, as defined in the Typology of Residential Buildings of Bosnia and Herzegovina for free-standing house, built in the interval from 1981. to 1991.

2. The values of optimal transit profiles defined according to the adaptive PSI method and the two-stage method are in the range of 8 cm to 13 cm of polystyrene insulation thickness. The lower value corresponds to the two-stage optimization, which includes both optimizations according to the techno-economic criteria in first stage, synthetic profile and current and desired state in second stage.

3. The conclusion itself about the thickness of the insulation of the external walls of free-standing houses is not a novelty, but what is important is that the PSI method has been calibrated very well in this way for application in decision-making and adoption of optimal rehabilitation measures.

4. Additionally, the authors adapted the method to incorporate standard and regulatory insulation measures for free-standing houses. By doing so, they recalibrated the proposed methods and validated the findings on optimal insulation thickness from a new perspective.

5. Another key conclusion is that insulation manufacturers often promote 'optimal' thicknesses primarily to maximize profits and energy savings. This study demonstrates that the actual optima are lower than the values typically advertised. Each optimization process is guided by specific criteria, seeking solutions that align with its particular objectives.

6. A key recommendation for using the PSI method, as well as other MCDM methods, is to employ criteria characterized by positive savings and non-zero values. This approach prevents computational errors and inconsistencies in the selection of optimal solutions. Consequently, applying standard linear normalization to the initial criteria values is sufficient to yield reliable results. Such an approach is well-suited for evaluating optimal rehabilitation measures and energy efficiency improvements in buildings.

According to the proposal from the paper [39] and Eq. (18), the optimal value of the thermal insulation thickness is calculated according to:

$$\text{xopt}=\sqrt{\frac{Cf\cdot HDD\cdot 24\cdot k\cdot PW}{1000\cdot C_i\cdot \eta}}-k\cdot R_{other}\approx 11.54\,\text{cm}. \quad (18)$$

Equation (18) contains the following variables: Cf=0.11 EUR/kWh, price of electricity, HDD=2800 heating degree days for Sarajevo, k=0.039 W/mK, thermal conductivity of the insulation material, PW=15 present worth factor, Ci=250 EUR/m^3 cost of installing 1 m^2 of thermal insulation divided with 0.1, η=0.98 efficiency of heating system, Rother=0.45 m^2W/K typical uninsulated wall, specific thermal resistance. All data for the variables correspond to Bosnia and Herzegovina and the Republic of Srpska. It can be concluded that the results obtained using adaptive PSI are well-validated by Eq. (18).

Table 11. Validation of the obtained optimal insulation result.

Validation approach	Main validation role	Obtained result
Classical PSI	Basic ranking reference	Improv.1 10 cm
Adaptive PSI	Iterative convergence and stability check	Optimal interval of approximately 8-13 cm
Entropy-weighted VIKOR	Independent MCDM comparison	Improv. 1 confirmed
Hasan's analytical life-cycle-cost model [39]	Engineering-economic benchmark	xopt = 11.54 cm

Table 11 summarizes the validation of the obtained optimal insulation result using both MCDM-based and analytical approaches. The comparison shows that the adaptive PSI framework converges toward an insulation interval that is consistent with the classical PSI ranking, the entropy-weighted VIKOR comparison, and Hasan's analytical life-cycle-cost model. The value of 11.54 cm obtained by the analytical model lies within the adaptive PSI interval of approximately 8-13 cm, confirming the techno-economic consistency of the proposed framework.

Similar integrated approaches combining performance evaluation with multi-criteria decision-making have been applied to assess engineering solutions based on environmental, economic, and functional indicators, leading to balanced and robust ranking outcomes [40], [41]. In a comparable manner, the results obtained in this study indicate that optimal alternatives emerge from the simultaneous consideration of multiple performance criteria rather than from the dominance of a single indicator. From a methodological perspective, the proposed PSI-based framework can be interpreted as a structured optimization process in which the criteria space is first stabilized through PSI evaluation and the construction of a synthetic decision profile, followed by the final ranking of alternatives. After that, optimal transit paths are formed between the observed states within the framework of adaptive optimization. This structure extends conventional MCDM approaches by introducing an additional decision layer that improves the logical consistency and robustness of the optimization outcomes.

7. Conclusion

This paper proposed an adaptive extension of the classical PSI method based on the construction of a synthetic profile. The main contribution is the transformation of the static PSI tool into an iterative, multi-criteria optimization framework that integrates data normalization selection, synthetic profile generation, data dispersion mitigation, and an embedded feedback loop. Within this process, PSI-derived participation coefficients are utilized to generate transitional optimization states. By defining strict convergence conditions toward a stable optimal alternative, this framework effectively enables and expands adaptive optimization capabilities via the PSI method.

The proposed framework was validated using a case study of optimal external wall insulation thickness for detached houses in Bosnia and Herzegovina. The results indicated a tendency of the adaptive PSI procedure toward insulation measures close to 10 cm of EPS insulation, while the generated synthetic profiles indicate an optimal techno-economic interval of approximately 8–13 cm. This result was further supported by comparison with an entropy-weighted VIKOR ranking and by an analytical life-cycle-cost insulation thickness formulation. The agreement with these reference approaches supports the practical relevance of the proposed framework, although broader validation is required.

The study also showed that the selection of the normalization method has a significant influence on the stability of PSI-based rankings. In particular, the adaptive PSI procedure can reveal rank instability caused by normalization effects and can therefore be used as a stability-checking mechanism in MCDM problems. The two-stage version of the method additionally demonstrated the possibility of coupling techno-economic optimization with scenario-based strategic planning.

Since the proposed framework was calibrated and demonstrated using a relatively small case study, the generalizability of the findings should be examined on larger and more diverse datasets, additional MCDM comparison methods, sensitivity analysis, and dynamic applications in which the synthetic profile could be updated in real time. Further development should also focus on defining formal convergence criteria and extending the adaptive PSI framework to multi-stage optimization problems. The novelty of the approach lies not in the mathematical form of the synthetic profile as a convex combination, but in its PSI-based derivation and its use as a feedback element within an adaptive MCDM process. From an engineering perspective, the method provides a transparent decision-support tool for identifying robust insulation ranges rather than relying only on a single optimal alternative.

Competing Interest Statement

The authors declare no known competing financial interests or personal relationships that could have influenced the work reported in this paper.

Data Availability Statement

Supplementary materials and data used in this research are accessible upon request. For access, please contact the corresponding author via [srdjan.vaskovic@ues.rs.ba].

Author Contributions

This paper was completed through the joint efforts of all authors, whose contributions are outlined below: Srđan Vasković, Petar Gvero, Đorđe Vojinović, Milovan Kotur, Ljubiša Preradović, Danijela Ančić-Kardaš, Milan Pupčević: Conceptualization, Methodology, Formal analysis, Investigation, Data curation, Writing–original draft, Writing–review & editing. Petar Gvero, Srđan Vasković: Supervision, Validation, Writing– review & editing.

Statement on the Ethical Use of AI Tools

AI tools were used for proofreading and improving linguistic quality. All corrections were manually reviewed and approved by the authors, who take full responsibility for the content.

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