

An Entropy–MARCOS Intelligent Decision Framework for Optimizing Cement-Stabilized Pond Ash in Geotechnical Applications

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Abstract

The reuse of industrial by-products in geotechnical engineering requires systematic optimization frameworks that balance multiple performance criteria simultaneously. Traditional mix design approaches for cement-stabilized pond ash rely primarily on heuristic evaluation of individual strength parameters, often neglecting integrated, data-driven decision support. This study proposes an artificial intelligence–assisted multi-criteria decision framework to identify the optimal cement content for pond ash stabilization. Experimental investigations were conducted to evaluate maximum dry density (MDD), optimum moisture content (OMC), and unconfined compressive strength (UCS) at 7, 14, and 28 days for six pond ash–cement mixtures. The resulting dataset was integrated into an objective weighting and ranking architecture that combines the Entropy method for automated weight derivation with the Measurement of Alternatives and Ranking according to COMPromise Solution (MARCOS) method for alternative prioritization. The Entropy algorithm identified early-age UCS as the most influential criterion, reflecting its high discriminative capacity within the dataset. The MARCOS-based ranking revealed that the 85:15 pond ash–cement proportion provides the optimal compromise, achieving balanced early- and long-term strength performance with acceptable compaction behavior. The proposed framework serves as a transparent, reproducible decision-support model for intelligent selection of geomaterials and demonstrates how artificial intelligence–assisted optimization can enhance sustainable geotechnical material design. The methodology is extensible to other industrial-waste stabilization systems, thereby supporting data-driven infrastructure engineering.

Keywords: Multi-criteria decision intelligence, Entropy weighting, MARCOS method, Cement-stabilized pond ash, Geotechnical optimization, Data-driven material selection.

1. Introduction

The large volume of ash produced by coal thermal power generation will, unsurprisingly, continue to be stored in wet ash ponds for the foreseeable future in many regions, including India. Storing ash in wet conditions (known as pond ash, or PA) imposes persistent pressure on land use, causes dust to become a nuisance when the material dries, and may introduce risks of leachate migration from the storage site, depending on local hydrogeology and ash chemistry. Meanwhile, the construction industry is experiencing increasing scarcity and rising costs of natural borrow materials for use in embankments, subgrades, and pavement layers. The interaction of these constraints creates a situation that encourages the valorization of industrial by-products such as PA into engineered geomaterials, provided that the material's inherent limitations (low cohesion, poor gradation, and insufficient strength gain under service moisture conditions) can be addressed through stabilization.

Cement stabilization remains the most widely practiced and supported method for improving weak soils and marginal geomaterials because it offers enhanced interparticle bonding, increased stiffness, and improved short-term constructability compared with un-stabilized conditions. However, cement stabilizing ash-based materials is not a task that only has a single objective; a dosage of cement that optimizes strength can also negatively affect workability, moisture demand, shrinkage potential, and carbon/cost intensity; conversely, the compaction response i.e. maximum dry density and optimum moisture content (MDD, OMC) of ash-based materials is influenced both by cement hydration and pozzolanic reactions that occur during field curing and/or moisture fluctuations [1]. Recent research on cemented and alternative binder systems has consistently shown that the type and dosage of stabilizer, curing duration, and moisture state of the material have a significant impact on the compaction-strength coupling and the durability performance [2], [3], [4].

A significant body of research describes the use of stabilized geomaterials in pavement and subgrade applications, primarily using strength parameter unconfined compressive strength (UCS), and compaction indicators MDD and OMC. A number of researchers have

identified systematic changes in compaction parameters and an increase in the strength of cement-ash-blended mixtures as they cure, which is often confirmed by microstructural analyses of the formation of cement bonding products [4], [5]. Equivalent trends have also been reported in other stabilized geomaterials, e.g., dredged sediments [6], [7], recycled aggregate composites [8], alternative binders for solid waste materials [9], [10]. There is also an increasing emphasis on measuring resilient characteristics in relation to stabilized base materials within pavement engineering applications [11], [12]. In addition to using traditional cement to stabilize ash-based materials, many researchers have also examined a range of low-carbon, waste-derived [13], and alternative binder systems [14], [15], including geopolymers, slag/fly-ash-composite, gypsum-modified, and/or the use of blended systems to reduce their environmental impact while achieving acceptable mechanical performance characteristics [16], [17].

Waste synergy approaches, e.g., adding incineration ashes, rubber, or supplementary cementitious materials, highlight that the "best" stabilization cannot be evaluated by a single performance metric, as hybrid materials may exhibit trade-offs among strength and ductility, workability and moisture demand, and sustainability objectives [18], [19]. In the case of PA in particular, the amount of water available to meet the hydration demand may become a critical control variable, particularly when the cement dosages are higher, and the "OMC from Proctor" (which represents adequate water) may not always correspond to "sufficient water to allow complete cement hydration" [20]. This indicates that when designing stabilization mixtures, it is necessary to consider the trade-off among short-term strength (i.e., to open the road to vehicle traffic), longer-term strength for serviceability, and compaction/moisture characteristics (constructability/stability in the field). As can be seen from the extent of experimental data, when evaluating the mixture(s) chosen for stabilization research, they are frequently described using a comparative summary of trends ("UCS increased with additive", "OMC increased/decreased with additive"), followed generally by an implied recommendation of dose. When multiple criteria are important simultaneously, the reliability of this approach declines (e.g., early UCS versus later UCS, or strength versus moisture demand).

A multi-criteria decision-making (MCDM) process is a logical, structured methodology for resolving multiple trade-off relationships; therefore, the first step is to assign criterion weights, and the second is to provide a clear ranking of alternatives. MCDM has been applied in civil engineering to support sustainability and scenario selection, thereby enabling the simultaneous assessment of engineering performance alongside the corresponding environmental and socio-economic metrics of each scenario [21].

Despite significant advancements in experimental soil stabilization research, the selection of optimal geomaterial mixtures remains largely heuristic and dependent on single-criterion performance indicators. Such conventional approaches fail to systematically capture the complex, multi-dimensional trade-offs among strength development, compaction behavior, and constructability constraints. In recent years, artificial intelligence and data-driven decision frameworks have emerged as powerful tools in civil engineering for transforming experimental datasets into structured optimization models [22], [23], [24]. Recent studies have further demonstrated the growing role of machine learning in pavement engineering for predicting material deterioration and supporting infrastructure management, highlighting the increasing adoption of data-driven methodologies in transportation engineering [25]. Unlike traditional empirical interpretation, Artificial intelligence-assisted multi-criteria systems enable automated weight learning, objective prioritization of performance indicators, and compromise-based ranking of alternatives. However, the integration of computational decision intelligence into geotechnical material optimization, particularly for industrial waste stabilization, remains limited [26]. Addressing this gap, the present study develops an Artificial intelligence-assisted multi-criteria decision framework that combines Entropy-based objective weight extraction with the Measurement of Alternatives and Ranking according to COmpromise Solution (MARCOS) ranking algorithm to enable transparent, reproducible, and data-driven optimization of cement-stabilized pond ash mixtures [27], [28], [29]. By bridging laboratory geomechanics with computational intelligence, this work advances the development of intelligent decision-support systems for sustainable infrastructure materials.

In the context of selecting a mixture of pond ash (PA) and cement, however, MCDM application has not been adequately explored based on the quantity of experimental studies conducted for the stabilization of PA, and thus an opportunity exists for the combination of laboratory performance data (i.e., relative measures of build-up quality) with objective ranking methodology to identify the optimal PA-cement stabilization mixture [30].

This study proposes selecting PA-cement mixtures based on empirical performance data collected across multiple PA/cement mixture proportions, framing the problem as a multi-criteria decision-making (MCDM) problem. Alternatives will be generated by treating each alternative PA-cement mixture as a discrete PA/cement blend, with a fixed PA-to-cement ratio. Then, each candidate PA/cement mixture will be assessed (i.e., ranked) based on empirical performance measures, compact ability, and strength, as measured by standard laboratory testing procedures. Specifically, laboratory testing will be conducted to establish the absolute compaction density (MDD) and optimum compaction moisture content (OMC) for compacted PA/cement mixtures, and both unconfined compressive strength (UCS) performance data will be collected at multiple curing intervals (7, 14, and 28-days) [31], providing an evaluation of the long-and short-term trends of cemented aggregates. The criteria will not be perfectly aligned, resulting in a relatively complex non-linear MCDM problem where, for example, a PA/cement mixture that generates the highest UCS will not necessarily generate a PA/cement mixture with the lowest compaction requirement, nor is it possible to establish a single ratio of PA/cement that can provide the maximum performance. As such, a well-defined decision framework must be used to identify the alternative mixture that best balances the selected criteria [32].

To address this MCDM problem, the present study employs a balanced entropy/MARCOS approach. The entropy approach provides objective (with respect to the data) weights for each criterion by analyzing the dispersion of performance measurements. This approach mirrors the principle of “letting the data speak for itself” and is particularly advantageous when expert consensus is low or expert opinions are unavailable. The MARCOS method assesses measured PA/cement mixtures based on their similarity to both ideal and anti-ideal solutions and

produces a clear, interpretable, compromise-based performance ranking system that can be used when engineers are required to select alternatives with multiple performance indicators (i.e., numerical values).

Recent studies have increasingly examined pond ash and related industrial by-products as sustainable geomaterials for pavement, subgrade, embankment, and lightweight fill applications. Earlier work demonstrated that combinations of pond ash, rice husk ash, cement, and fiber can improve the compaction and strength behaviour of clayey soils, supporting their use in embankments and reinforced earth structures [33]. Gupta and Kumar [30] showed that pond ash–rice husk ash–cement mixtures can satisfy highway material requirements, with compaction behaviour, California bearing ratio, and microstructural characteristics confirming their engineering suitability. Sudheer Kumar and Sharma reported that pond ash, mixed with cement kiln dust and polypropylene fiber, improved density, bearing capacity, and shear strength, indicating its potential for use in highway embankments and structural fill [31].

Recent studies have broadened the assessment of pond ash stabilization beyond traditional strength tests. Jose, et al. explored cement-stabilized pond ash under conditions of resilient and permanent deformation, demonstrating that even minimal cement additions can render pond ash suitable for compacted pavement subgrades [32]. Building on this, Jose, et al. [33] analysed repeated-loading behaviour using the shakedown concept, highlighting the significance of curing, moisture content, compaction, and cement dosage for pavement performance. Pande and Padade [34] created a pond ash geomaterial with expanded polystyrene beads, cement, and water, illustrating its potential as a lightweight fill. Varma and Singh [35] compared geopolymer-, cement-, and lime-treated pond ash, finding that geopolymer-treated pond ash offers superior strength and durability for pavement applications.

In parallel, optimization and data-driven approaches have begun to gain attention in the design of pond ash- and waste-based geomaterials. Dev, et al. [36] applied machine learning models to predict the unconfined compressive strength of controlled low-strength material containing fly ash and pond ash, identifying cement content, water content, flowability, pond ash content, and

curing period as important predictors. P. P. and Thangasamy [37] also optimized pond ash–lime kiln dust mixtures to improve expansive soil using response surface methodology. P. P., et al. [38] used response surface methodology for long-term lime and cement stabilization of pond ash, showing that additive content and curing period strongly influence strength development. These studies demonstrate the growing relevance of computational and optimization-based approaches in sustainable geomaterial selection.

However, most existing studies focus on experimental characterization, statistical prediction, or single-objective optimization. Limited attention has been given to the objective, multi-criteria ranking of pond ash–cement mixtures that simultaneously considers compaction behaviour and multi-age strength performance. This motivates the present study, which integrates experimental geotechnical testing with an Entropy–MARCOS decision framework to identify the most balanced pond ash–cement mixture. The paper is organized as follows: Section 2 presents material collection, experimental testing, and the Entropy–MARCOS methodology; Section 3 discusses compaction, strength, and decision-ranking results; and Section 4 summarizes the conclusions, limitations, and future research directions.

The objective of this study is to develop a clearly defined workflow for evaluating PA–cement stabilization alternatives using MCDM. In more detail, the present study's objectives are as follows:

- Define candidate PA cement mixtures based on empirical performance data. Establish MDD and OMC through standard Proctor testing to quantify the compact ability of candidate PA/cement mixtures.
- Evaluate both short and long-term performance of candidate PA/cement mixtures based on UCS testing conducted at multiple time intervals (7, 14, and 28-days) to characterize the expected trends in cemented aggregates.
- Apply the Entropy method to derive objective performance measures for the ranking of each alternative candidate PA/cement mixture.
- Employ the MARCOS approach to rank candidate PA/cement mixtures and establish the optimal

(compromise) PA/cement mixture based upon compact ability and UCS criteria.

- Develop a decision-support system that defines a transparent and repeatable decision-support process for selecting PA/cement alternatives; thereby allowing for future extensions to other waste-stabilized material applications (blended binder systems, recycled aggregate).

The principal contributions of this study are as follows:

- **Engineering Application:** The results of this study will establish a rational methodology for assessing the short-term and long-term performance (i.e., compact ability and UCS) of PA/cement mixtures when constructing pavements and other civil engineering structures, thus providing engineers with the tools necessary to achieve their project goals based on measurable performance metrics for PA-cement mixtures. Engineering-based performance metrics for PA-cement mixtures consistently underscore the need for performance-based methods in the civil engineering community for the construction of both base and subgrade structures.
- **Decision-Making Transparency:** By integrating the Entropy method with the MARCOS approach, the present study establishes a decision-making process that provides for transparent decision-making by transforming both multi-aged UCS and compact ability empirical results into a clear (i.e., auditable) ranking of PA/cement alternatives, as opposed to relying on subjective/descriptive "best dosage narratives". This decision-making process is consistent with the objective-based assessment trends associated with sustainable and resilient infrastructure planning.
- **Transferability:** The methodology employed for MCDM-based evaluations of PA/cement alternatives in the present study is not limited to establishing optimal PA/cement alternatives; it can also be applied to establishing optimal alternatives for other industrial residues or partial cementitious materials.

2. Materials and Methods

This section describes the experimental materials, mix preparation procedure, testing program, and computational decision-making framework adopted to evaluate and rank the cement-stabilized pond ash mixtures.

2.1. Material collection (experimental data)

2.1.1. Pond ash

The pond ash (PA) used in the present study was collected from the ash pond of the Guru Gobind Singh Super Thermal Power Project (GGSSTP), Ropar, Punjab, India. PA is an abundant industrial by-product of coal combustion and exhibits cementitious behavior owing to its mineralogical composition. The chemical composition of PA, as reported in the thermal power plant's official records, indicates that it is predominantly composed of silica (SiO_2) and alumina (Al_2O_3), which together contribute significantly to its pozzolanic reactivity.

The physical properties of PA were determined in the laboratory in accordance with relevant Indian Standard procedures (IS 3025 series). The material was observed to be non-plastic, fine-grained, and to exhibit moderate shear strength and permeability, rendering it unsuitable for direct geotechnical applications without stabilization. The detailed chemical and physical properties of PA are presented in Table 1.

Table 1. Chemical and physical properties of pond ash.

Chemical composition (%)		Physical properties	
Loss of Ignition	4.52	Specific gravity	2.1
Silica (SiO_2)	56.32	Plasticity	Non-plastic
Alumina (Al_2O_3)	30.87	Maximum dry density (MDD)	11.01 kN/m^3
Iron oxide (Fe_2O_3)	4.94	Optimum moisture content (OMC)	27.40%
Magnesium oxide (MgO)	1.58	Angle of internal friction (ϕ)	33°
Calcium oxide (CaO)	0.7	Cohesion	1 kN/m^2
		Permeability	$1.24 \times 10^{-4} \text{ cm/s}$
		Coefficient of uniformity (Cu)	8.56
		Coefficient of curvature (Cc)	1.41

2.1.2. Ordinary Portland Cement

Ordinary Portland Cement (OPC) of 53 grade, ACC brand, was used as the stabilizing agent in this investigation. The cement was procured from the local market and conformed to the requirements specified in the relevant Indian Standards (IS 12269). OPC 53 grade was selected for its rapid early strength development and consistent performance in soil and ash stabilization applications.

Table 2. Chemical composition and physical requirements of OPC 53 grade cement.

Chemical composition (%)	Physical requirements
Lime (CaO)	62–67
Silica (SiO ₂)	17–25
Alumina (Al ₂ O ₃)	3–8
Calcium sulphate (CaSO ₄)	3–4
Iron oxide (Fe ₂ O ₃)	3–4
Magnesium oxide (MgO)	0.1–3
Sulphur (S)	1–3
Alkalies	0.2–1
Fineness	≥ 225 m ² /kg
Soundness (Le-Chatelier)	≤ 10 mm
Initial setting time	≥ 30 min
Final setting time	≤ 600 min
Compressive strength (72 ± 1 h)	≥ 27 MPa
Compressive strength (168 ± 2 h)	≥ 37 MPa
Compressive strength (672 ± 4 h)	≥ 53 MPa

The cement met the standard physical requirements for fineness, soundness, setting time, and compressive

strength at different curing ages. Manufacturer-supplied data indicate that the cement is rich in lime (CaO) and silica (SiO₂), which govern hydration reactions and strength development. The chemical composition and key physical requirements of OPC are summarized in Table 2.

2.1.3. Mix proportions and experimental testing

PA and OPC were thoroughly mixed in dry conditions to obtain a homogeneous blend. Six different PA–cement mix proportions were prepared by replacing PA with cement at incremental percentages by dry weight.

Standard Proctor compaction tests were conducted to determine the MDD and OMC of each mix. Cylindrical specimens were prepared at their respective OMC and MDD and subjected to UCS testing after 7, 14, and 28 days of curing. All experimental investigations were carried out in accordance with relevant Indian Standard Specifications (IS 2720 Part 7 & 8). The combined compaction and strength test results obtained from this experimental program constitute the quantitative dataset used for subsequent multi-criteria decision-making analysis. The overall workflow of material collection, characterization, mix proportioning, and specimen preparation adopted in this study is schematically illustrated in Figure 1. The framework highlights the systematic transformation of raw industrial by-products into structured experimental datasets for subsequent multi-criteria decision-making analysis.

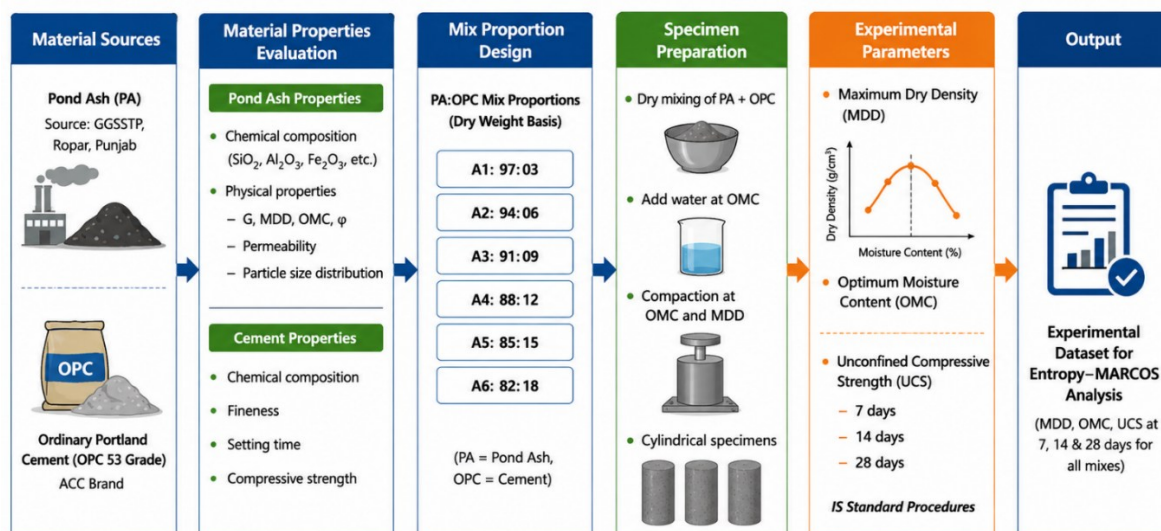


Figure 1. Schematic representation of material collection, characterization, mix proportioning, specimen preparation, and experimental testing of pond ash–cement mixtures.

2.2. Multi-Criteria Decision-Making Methodology (Entropy–MARCOS)

To select the optimal pond ash–cement mix while balancing multiple, often conflicting performance indicators, the study applied an MCDM framework integrating the Entropy method for objective weighting and the MARCOS method for ranking [39]. Entropy assigns weights based on data variability [40], [41], [42], while MARCOS ranks alternatives by their proximity to ideal and anti-ideal solutions [43], [44].

The six PA–cement mixtures were treated as discrete alternatives because each mix represents a feasible stabilization option with a different cement replacement level. The alternatives A1–A6 correspond to PA: Cement proportions of 97:03, 94:06, 91:09, 88:12, 85:15, and 82:18, respectively. These proportions were selected to evaluate the progressive influence of cement content on compaction behaviour and strength development.

The evaluation criteria were selected based on their direct relevance to geotechnical and pavement material performance. MDD was considered because it reflects the achievable compacted density and packing efficiency of the stabilized mix. OMC was included because it governs field moisture requirement and workability during compaction. UCS at 7, 14, and 28 days was selected to capture early-, intermediate-, and late-age strength development, respectively. MDD and UCS were treated as benefit criteria, as higher values indicate better density and strength performance, whereas OMC was treated as a cost criterion because lower moisture demand is generally preferred for field compaction and constructability.

The Entropy method was selected because it provides objective criteria weights based on the dispersion of experimental data, thereby reducing subjectivity in assigning importance to performance indicators. Optimization-based computational frameworks have increasingly been employed to solve complex engineering decision problems involving multiple, often conflicting, design objectives. Such approaches improve the transparency and reproducibility of engineering decision-making across infrastructure applications [45]. MARCOS was selected because it ranks alternatives by comparing each option with ideal and anti-ideal solutions, making it suitable for compromise-based selection when multiple criteria conflicts. Therefore, the combined Entropy–

MARCOS framework was considered appropriate for identifying the most balanced PA–cement mix from the experimental dataset.

2.3. Decision matrix formulation

Consider m alternatives A_i ($i = 1, 2, \dots, m$), representing pond ash–cement mixes, and n criteria C_j ($j = 1, 2, \dots, n$), including MDD, OMC, and UCS at 7, 14, and 28 days. The decision matrix is as per Equation (1):

$$\mathbf{X} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \quad (1)$$

Six PA–cement mixes form the alternatives. Benefit criteria (MDD, UCS) are higher-is-better; cost criteria (OMC) are lower-is-better.

2.3.1. Entropy-based criteria weighting

Entropy objectively weights criteria by their information dispersion. First, normalize the matrix to proportions, Equation (2). Entropy for criterion j is as per Equation (3), diversification degree by Equation (4), and weights by Equation (5). Higher-variability criteria receive greater weights, reflecting their influence on the decision [46].

$$p_{ij} = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}}, \quad \sum_{i=1}^m p_{ij} = 1 \quad (2)$$

$$E_j = -k \sum_{i=1}^m p_{ij} \ln(p_{ij}), \quad (3)$$

$$k = \frac{1}{\ln m}$$

$$d_j = 1 - E_j \quad (4)$$

$$w_j = \frac{d_j}{\sum_{j=1}^n d_j}, \quad (5)$$

$$\sum_{j=1}^n w_j = 1$$

2.3.2. MARCOS method

MARCOS ranks alternatives by distances to the ideal (AI) and anti-ideal (AAI) solutions [47].

Step 1: Extended Matrix

Append ideal and anti-ideal rows (Equation (6)).

$$\mathbf{X}^* = \begin{bmatrix} x_{AAI,1} & \cdots & x_{AAI,n} \\ x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \\ x_{AI,1} & \cdots & x_{AI,n} \end{bmatrix} \quad (6)$$

For benefit criteria: $x_{(AI,j)} = \llbracket \max \rrbracket_i x_{ij}$, $x_{(AAI,j)} = \llbracket \min \rrbracket_i x_{ij}$.

For cost criteria: $x_{(AI,j)} = \llbracket \min \rrbracket_i x_{ij}$, $x_{(AAI,j)} = \llbracket \max \rrbracket_i x_{ij}$.

Step 2: Normalization by Equation (7)

$$n_{ij} = \begin{cases} \frac{x_{ij}}{x_{AI,j}} & (\text{benefit}) \\ \frac{x_{AI,j}}{x_{ij}} & (\text{cost}) \end{cases} \quad (7)$$

Step 3: Weighted Normalized Matrix by Equation (8)

$$v_{ij} = w_j \cdot n_{ij} \quad (8)$$

Step 4: Utility Functions

Utility scores: $S_i = \sum_{j=1}^n v_{ij}$, with S_{AAI} and S_{AI} for anti-ideal and ideal. Relative utilities are defined in Equation (9).

$$K_i^- = \frac{S_i}{S_{AAI}}, K_i^+ = \frac{S_i}{S_{AI}} \quad (9)$$

Step 5: Ranking

Final utility by Equation (10).

$$f(K_i) = K_i^+ = \frac{S_i}{S_{AI}} \quad (10)$$

Rank alternatives descending by $f(K_i)$; higher values indicate better performance.

This Entropy–MARCOS approach systematically evaluates mix designs using compaction and strength data

to identify the optimal balance between performance and efficiency. Weights and rankings are detailed in the Results section.

2.4. Intelligent decision framework architecture

To operationalize the integration of experimental geomechanics with artificial intelligence–assisted decision modeling, a structured computational framework was developed, as illustrated conceptually in Figure 2. The architecture consists of four sequential layers:

- **Data Acquisition Layer:** Experimental inputs, including MDD, OMC, and UCS at 7, 14, and 28 days, are collected from laboratory testing. This structured dataset forms the quantitative basis for intelligent evaluation.
- **Feature Normalization and Weight Learning Layer:** The raw experimental matrix is normalized to eliminate dimensional inconsistencies across criteria. The Entropy algorithm automatically determines the relative importance of each criterion based on information dispersion, enabling objective, data-driven weight extraction without subjective bias.
- **Decision Intelligence Engine (MARCOS Module):** The normalized and weighted matrix is processed through the MARCOS compromise-ranking algorithm. This module constructs ideal and anti-ideal solutions, computes utility degrees, and generates aggregated performance indices for each alternative mix proportion.
- **Optimal Solution and Decision Output Layer:** Final utility scores are used to rank mix alternatives, identifying the optimal compromise solution that balances strength development and compaction behavior. The framework provides transparent, reproducible decision-support output suitable for engineering design.

This layered architecture demonstrates how experimental geotechnical datasets can be transformed into an intelligent decision-support system through algorithmic weight learning and structured alternative ranking. The framework is extensible and can incorporate additional performance criteria (e.g., durability, cost, carbon footprint) or machine learning modules for predictive optimization in future studies.

The proposed framework is considered AI-assisted because it transforms experimental datasets into an automated decision-support system by learning the importance of criteria and optimizing objectives across competing alternatives. Although Entropy and MARCOS are classical computational intelligence techniques rather than machine learning algorithms, they constitute explainable Artificial intelligence-inspired decision analytics by replacing subjective expert judgement with data-driven inference. The framework therefore represents an interpretable intelligent decision-support architecture suitable for engineering optimization.

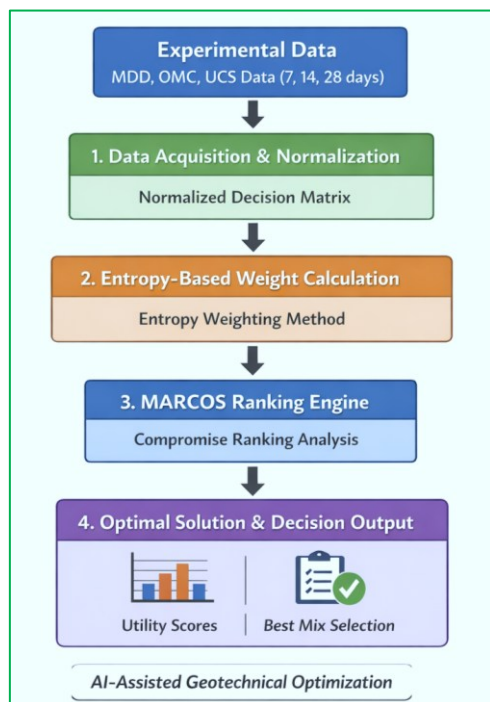


Figure 2. Computational AI-based decision-support architecture integrating experimental data acquisition, Entropy-based objective weighting, MARCOS ranking, and optimal mixture selection.

To enhance transparency and reproducibility, all computational analyses, including Entropy weighting,

MARCOS ranking, sensitivity analysis, comparative validation, and automated result generation, were implemented in Python. The complete source code together with the generated output tables is provided in the Supplementary Material (Appendix).

3. Results and Discussion

The results obtained from the laboratory investigations conducted on pond ash stabilized with varying proportions of cement are presented and discussed in this section. The experimental program included determining specific gravity (IS 2720 Part 3), grain-size distribution (IS 2720 Part 4), compaction characteristics (IS 2720 Part 7 & 8), and UCS (IS 2720 Part 10). Table 3 indicates all results and the decision matrix.

3.1. Specific gravity test

The specific gravity of pond ash was determined using the standard relation:

$$G = \frac{(W_2 - W_1)}{(W_2 - W_1) - (W_3 - W_4)}$$

where,

W_1 = weight of empty pycnometer,

W_2 = weight of pycnometer + dry sample,

W_3 = weight of pycnometer + sample + water,

W_4 = weight of pycnometer + water.

Substituting the measured values:

$$G = \frac{(688 - 488)}{(688 - 488) - (1374 - 1269)} = 2.10$$

The specific gravity of 2.10 indicates that pond ash has a relatively low unit weight compared to natural soils, a characteristic of coal combustion by-products that influences its compaction and strength behavior.

Table 3. Mix proportions and experimental results (Decision Matrix for MCDM).

Alternatives	Mix ID (PA: Cement)	Cement (%)	MDD (g/cm ³)	OMC (%)	UCS 7 days (N/mm ²)	UCS 14 days (N/mm ²)	UCS 28 days (N/mm ²)
A1	97:03	3	1.135	28.2	0.524	0.796	0.97
A2	94:06	6	1.14	28	0.595	0.97	1.066
A3	91:09	9	1.193	27.8	0.696	1.198	1.338
A4	88:12	12	1.194	27	0.872	1.254	1.371
A5	85:15	15	1.196	25.8	1.185	1.219	1.587
A6	82:18	18	1.239	24	1.006	1.425	1.588

3.2. Grain size analysis

Grain size analysis was performed on pond ash to assess its particle-size distribution. The particle size distribution curve is shown in Figure 3. From the curve, the following characteristic particle sizes were obtained:

$$D_{10} = 0.0066 \text{ mm}$$

$$D_{30} = 0.023 \text{ mm}$$

$$D_{60} = 0.0565 \text{ mm}$$

The coefficients of gradation were calculated as:

$$C_u = \frac{D_{60}}{D_{10}} = \frac{0.0565}{0.0066} = 8.56$$

$$C_c = \frac{(D_{30})^2}{D_{10} \times D_{60}} = \frac{(0.023)^2}{0.0066 \times 0.0565} = 1.41$$

The values $C_u > 4$ and $1 < C_c < 3$ indicate that the pond ash is well-graded, with particles of different sizes in favorable proportions. Such gradation is beneficial for compaction and strength development when used in stabilized mixes.

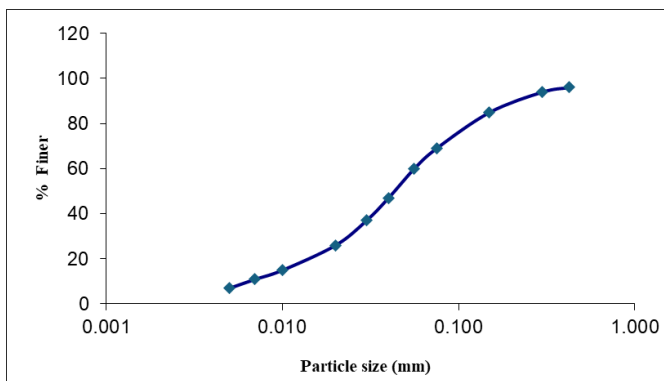


Figure 3. Particle size distribution curve.

3.3. Standard Proctor compaction test

MDD and OMC were determined for six different pond ash-cement mixing proportions using standard Proctor compaction tests. Figures 4 and 5 illustrate the variation of MDD and OMC with cement content. As shown in Figure 4, increasing the cement content further increases the MDD. At a cement content of 3%, the MDD is approximately 1.135 g/cm³, whereas at 18% it is 1.239 g/cm³. It can be observed that, the addition of cement will increase the MDD due to possible densification of the

pond ash matrix. Figure 5 shows how OMC decreases when the amount of cement increases. The OMC was approximately 28.2% at the lowest cement percentage and approximately 24% at the highest. Therefore, cement-stabilized mixes require less moisture because the particles are more tightly bonded and the void ratio is lower.

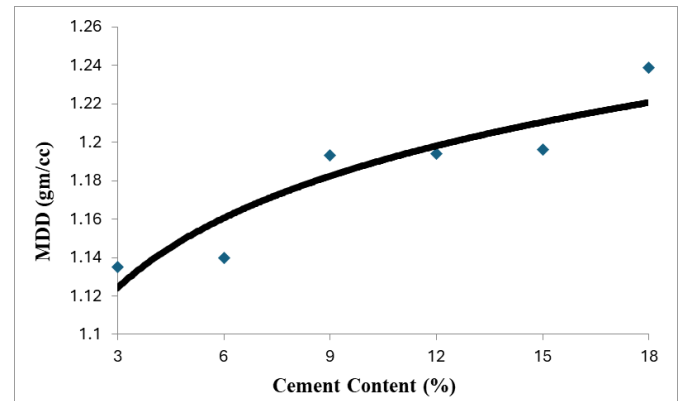


Figure 4. Variation of maximum dry density with cement content for pond ash-cement mixtures.

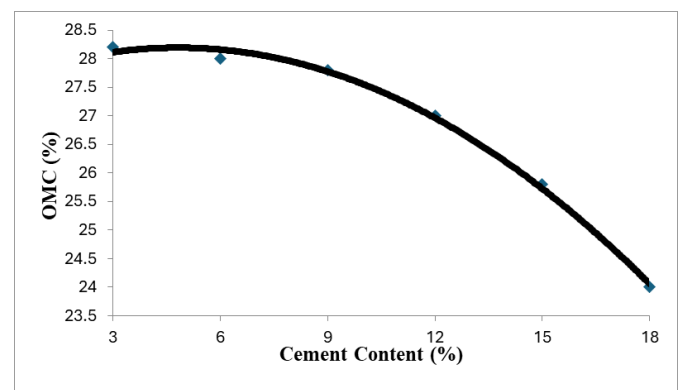


Figure 5. OMC versus cement content.

The compaction behavior indicates that increasing cement content improves packing efficiency and reduces moisture demand, which is favorable for field compaction and subsequent strength development.

3.4. Unconfined compressive strength

To assess the time-dependent strength gain of pond ash (PA)-cement mixtures, UCS tests were performed on mixes cured for 7, 14, and 28 days. UCS has been shown to be a useful measure for evaluating the effectiveness of stabilization procedures as defined in IS 4332 (Part 5). The stress-strain response for all mixes is shown in Figures 6-11. These figures represent the range of cement

(C) percentages in the mixes, from 3% to 18%. Stress and stiffness increased as curing progressed and as the mix's C content increased. Lower percentages of C produced flatter stress-strain curves and lower peak stresses, whereas the addition of C produced steeper stress-strain curves and a greater resistance to axial deformation. UCS increases with C at each curing period (Table 3), therefore, this supports the previously explained course of cement hydration products, which contribute to strength gain. The peak UCS for the mix comprising 82:18 PA: Cement was observed after 28 days of curing, while the mix comprising 97:03 PA: Cement had the lowest strength during the earlier curing periods. The gradual increase in UCS during additional curing confirms ongoing cement hydration and the formation of pozzolanic products through the reaction of cement with PA, which enhance bond strength and load-carrying capacity. Better stress-strain behavior at later curing ages provides additional evidence of increased stiffness and structural integrity in the stabilized mixes.

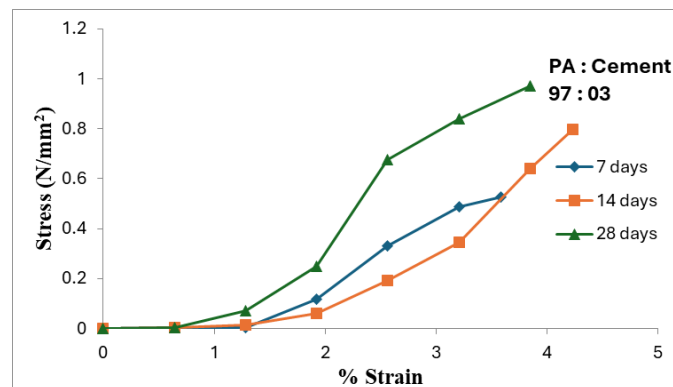


Figure 6. Variations in stress-strain of a sample (97:03) at 7, 14, and 28 days of curing.

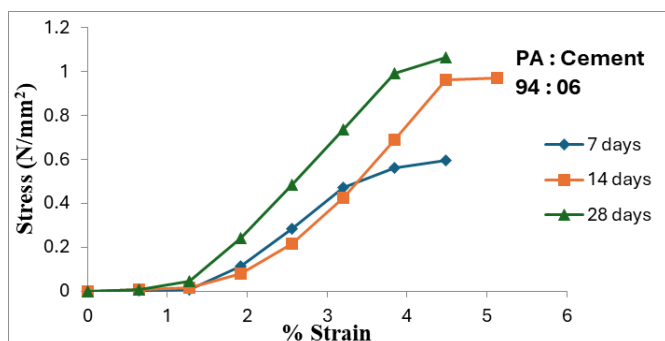


Figure 7. Variations in stress-strain of a sample (94:06) at 7, 14, and 28 days of curing.

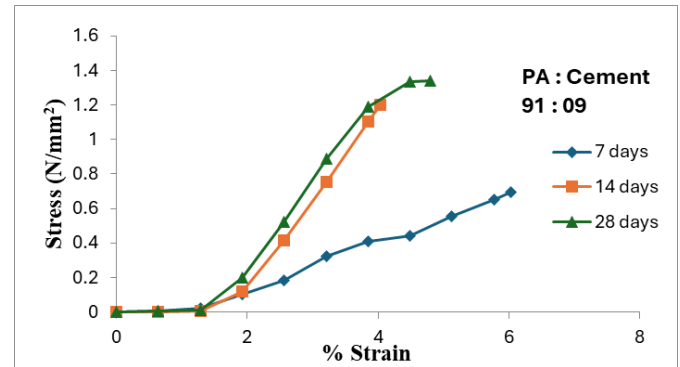


Figure 8. Variations in stress-strain of a sample (91:09) at 7, 14, and 28 days of curing.

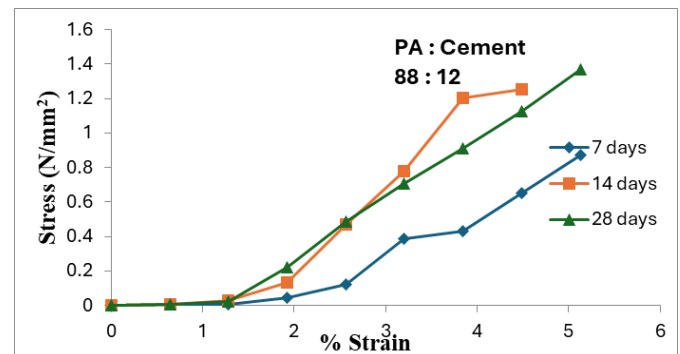


Figure 9. Variations in stress-strain of a sample (88:12) at 7, 14, and 28 days of curing.

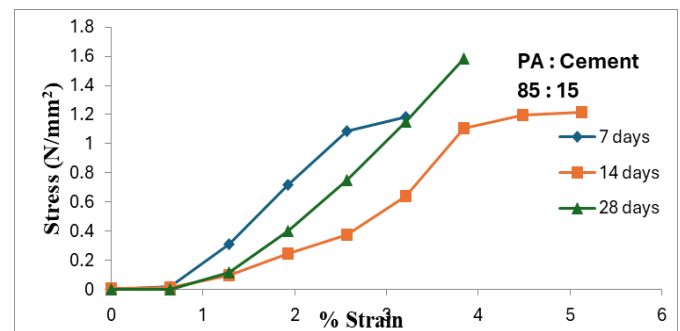


Figure 10. Variations in stress-strain of a sample (85:15) at 7, 14, and 28 days of curing.

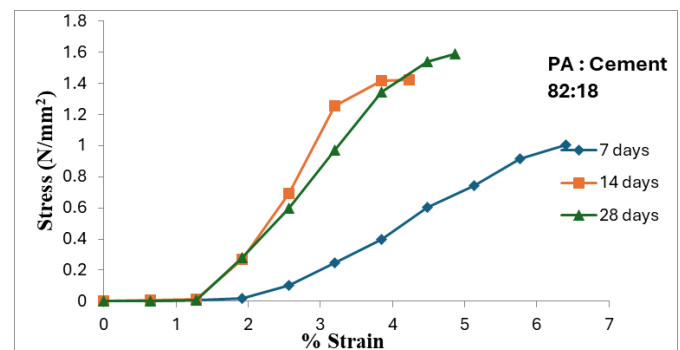


Figure 11. Variations in stress-strain of a sample (82:18) at 7, 14, and 28 days of curing.

Increasing cement content results in higher dry density and lower moisture requirement, as evidenced by compaction behavior. UCS improves consistently with both cement content and curing duration. Higher cement proportions exhibit stiffer stress–strain responses and greater peak strength. While the experimental results clearly demonstrate performance trends, no single mix is ranked at this stage, as multiple criteria must be considered simultaneously.

3.5. Application of MCDM

After preparing the Entropy–MARCOS method as explained in Section 2.2 and 2.3, experimental compaction and strength test data have been used to rank the pond ash–cement mixes. The MCDM procedure followed is systematic, starting with: (i) Entropy-based normalization and weight of criteria, (ii) Normalization and utility assessment by MARCOS, (iii) Ranking and interpretation of alternatives. The complete computational workflow, including Python implementations of Entropy weighting, MARCOS ranking, sensitivity analysis, comparative validation using additional MCDM methods, and automated generation of result tables, is provided in the Supplementary Material (Appendix) to facilitate full reproducibility of the proposed framework.

3.5.1. Entropy-based normalization and criteria weighting

The first decision matrix containing six alternatives A1–A6 (Table 3), equal to PA: Cement mix proportions 97:03 to 82:18, and five evaluation criteria (MDD, OMC, and UCS at 7, 14, and 28 days) was normalized using Equation (2). The normalized decision matrix, which measures the contribution of each alternative to each criterion, is shown in Table 4. Table 4 demonstrates that the normalized UCS values increase noticeably with cement content (more pronounced for early-age strength (7 days)), whilst the normalized MDD values display marginal variation, and the OMC values decrease with the addition of cement.

Using the normalized matrix (Table 4), the entropy values E_j for each criterion were calculated using Equation (3). The corresponding degrees of

diversification d_j were obtained using Equation (4), and the final entropy weights w_j were computed using Equation (5). These results are summarized in Table 5.

Table 4. Normalized Decision Matrix for the Entropy Method.

Alternatives	PA: Cement	MD D	OM C	UCS (7 days)	UCS (14 days)	UCS (28 days)
A1	97:03	0.1599	0.1754	0.1074	0.116	0.1225
A2	94:06	0.1606	0.1741	0.122	0.1414	0.1346
A3	91:09	0.1681	0.1729	0.1427	0.1746	0.1689
A4	88:12	0.1682	0.1679	0.1788	0.1827	0.1731
A5	85:15	0.1685	0.1604	0.2429	0.1776	0.2004
A6	82:18	0.1746	0.1493	0.2062	0.2077	0.2005

Table 5. Entropy values, degree of diversification, and final weights of evaluation criteria.

Criterion	Entropy	Degree of Diversification	Entropy Weight
MDD	0.9997	0.0003	0.0061
OMC	0.9991	0.0009	0.0208
UCS (7 days)	0.9774	0.0226	0.5369
UCS (14 days)	0.9907	0.0093	0.2201
UCS (28 days)	0.9909	0.0091	0.2162

From Table 5, UCS at 7 days received the highest weight (0.5369), followed by UCS at 14 days (0.2201) and UCS at 28 days (0.2162), indicating that strength-related parameters dominate the decision-making process due to their higher variability among alternatives. In contrast, MDD (0.0061) and OMC (0.0208) were assigned relatively low weights, reflecting their limited dispersion.

3.5.2. MARCOS application

The MARCOS analysis was initiated by constructing the extended decision matrix, which includes the anti-ideal (AAI) and ideal (AI) solutions, as defined in Section 2.3.2.

Table 6. Extended Decision Matrix and Corresponding MARCOS Normalized Values.

Alternative	MDD	OMC	UCS (7 d)	UCS (14 d)	UCS (28 d)	MDD (Norm.)	OMC (Norm.)	UCS (7 d) (Norm.)	UCS (14 d) (Norm.)	UCS (28 d) (Norm.)
AAI (Anti-ideal)	1.135	28.2	0.524	0.796	0.97	0.9161	0.8511	0.4422	0.5586	0.6108
A1	1.135	28.2	0.524	0.796	0.97	0.9161	0.8511	0.4422	0.5586	0.6108
A2	1.14	28	0.595	0.97	1.066	0.9201	0.8571	0.5021	0.6807	0.6713
A3	1.193	27.8	0.696	1.198	1.338	0.9629	0.8633	0.5873	0.8407	0.8426
A4	1.194	27	0.872	1.254	1.371	0.9637	0.8889	0.7359	0.88	0.8634
A5	1.196	25.8	1.185	1.219	1.587	0.9653	0.9302	1	0.8554	0.9994
A6	1.239	24	1.006	1.425	1.588	1	1	0.8489	1	1
AI (Ideal)	1.239	24	1.185	1.425	1.588	1	1	1	1	1

For the benefit criteria (MDD and UCS), the ideal solution is the maximum observed value, whereas for the cost criterion (OMC), it is the minimum observed value. The extended decision matrix and the corresponding normalized values, computed using Equation (7), are presented in Table 6.

Subsequently, the weighted normalized matrix was obtained by multiplying the normalized values by their respective entropy weights, as given in Equation (8). The weighted normalized values for each alternative, along with the AI and AAI solutions, are reported in Table 7.

Table 7. Weighted normalized matrix (MARCOS).

Alternative	MDD	OMC	UCS (7 d)	UCS (14 d)	UCS (28 d)
AAI	0.0056	0.0177	0.2374	0.1229	0.1321
A1	0.0056	0.0177	0.2374	0.1229	0.1321
A2	0.0056	0.0178	0.2696	0.1498	0.1451
A3	0.0059	0.0180	0.3153	0.1850	0.1822
A4	0.0059	0.0185	0.3951	0.1937	0.1867
A5	0.0059	0.0193	0.5369	0.1883	0.2161
A6	0.0061	0.0208	0.4558	0.2201	0.2162
AI	0.0061	0.0208	0.5369	0.2201	0.2162

The aggregated utility score S_i for each alternative was then calculated by summing the weighted normalized values. The utility degrees relative to the anti-ideal solution K_i^- and the ideal solution K_i^+ were computed using Equation (9). Finally, the overall utility function $f(K_i)$, calculated using Equation (10), was used to rank the alternatives. The aggregated scores, utility degrees, final utility values, and rankings are presented in Table 8.

Table 8. MARCOS Aggregated Scores, Utility Degrees, and Final Ranking.

PA: Cement	Aggregated Score	Utility Degree (Relative to AAI)	Utility Degree (Relative to AI)	Final Utility	Rank
A1	0.5157	1	0.5157	0.5157	6
A2	0.588	1.1401	0.5879	0.5879	5
A3	0.7064	1.3697	0.7063	0.7063	4
A4	0.7998	1.5509	0.7997	0.7997	3
A5	0.9665	1.8741	0.9664	0.9664	1
A6	0.919	1.782	0.9189	0.9189	2

3.5.3. Sensitivity and comparative validation

To examine the robustness of the Entropy–MARCOS ranking, a sensitivity analysis was performed by varying the most influential criterion, UCS at 7 days, by $\pm 10\%$ and $\pm 20\%$. The remaining criterion weights were proportionally adjusted to ensure the total weight equaled 1. In addition, an equal-weight scenario was considered to evaluate whether the ranking is strongly dependent on the entropy-derived weight structure.

The results of the sensitivity analysis are presented in Table 9. Under the base Entropy weights, A5 obtained the highest utility value of 0.9664 and ranked first, followed by A6 and A4. When the 7-day UCS weight was decreased by 20% and 10%, A5 remained the best-ranked alternative, with utility values of 0.9586 and 0.9625, respectively. Similarly, when the 7-day UCS weight was increased by 10% and 20%, A5 continued to rank first, with utility values of 0.9703 and 0.9742, respectively. These results indicate that moderate variation in the dominant criterion weight does not change the optimal alternative. However, in the equal-weight scenario, A6

ranked first, followed by A5 and A4, indicating that the final ranking may shift when all criteria are given equal weight. This confirms that the Entropy-based weighting scheme plays an important role in reflecting the actual discriminatory power of the experimental dataset.

Table 9. Sensitivity analysis of MARCOS ranking under weight variation.

Scenario	Best alternative	Top-three ranking	Observation
Base Entropy UCS 7 d	A5	A5 > A6 > A4	A5 remains rank 1
-20% UCS 7 d	A5	A5 > A6 > A4	A5 remains rank 1
-10% UCS 7 d	A5	A5 > A6 > A4	A5 remains rank 1
+10% UCS 7 d	A5	A5 > A6 > A4	A5 remains rank 1
+20% UCS 7 d	A5	A5 > A6 > A4	A5 remains rank 1
Equal weights	A6	A6 > A5 > A4	Ranking changes under equal weighting

A comparative validation was also conducted using five additional MCDM methods: Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), Complex Proportional Assessment (COPRAS), Evaluation Based on Distance from Average Solution (EDAS), Weighted Aggregated Sum Product Assessment (WASPAS), and VIšeKriterijumska Optimizacija I Kompromisno Rješenje (VIKOR). The purpose of this comparison was to determine whether the MARCOS-derived ranking is consistent with other established decision-making approaches. As shown in Table 10, all six methods produced the same ranking order: A5 > A6 > A4 > A3 > A2 > A1. This confirms that A5 is consistently identified as the best compromise solution and that the proposed Entropy–MARCOS result is not method-specific. The agreement among different MCDM methods strengthens the reliability of the decision and supports selecting the 85:15 PA: Cement mixture as the optimal stabilized pond ash proportion.

The sensitivity and comparative validation results demonstrate that the selected optimal mix is robust under moderate weight variation and consistent across multiple MCDM methods. Therefore, the Entropy–MARCOS framework provides a reliable basis for ranking cement-stabilized pond ash mixtures. Nevertheless, the results may change if additional criteria such as durability,

resilient modulus, permeability, shrinkage, leachability, embodied carbon, or material cost are included. Future studies may extend the present framework by incorporating these criteria to support broader techno-environmental optimization of stabilized geomaterials.

Table 10. Comparative MCDM validation summary.

Method	Rank 1	Rank 2	Rank 3	Ranking order
MARCOS	A5	A6	A4	A5 > A6 > A4 > A3 > A2 > A1
TOPSIS	A5	A6	A4	A5 > A6 > A4 > A3 > A2 > A1
COPRAS	A5	A6	A4	A5 > A6 > A4 > A3 > A2 > A1
EDAS	A5	A6	A4	A5 > A6 > A4 > A3 > A2 > A1
WASPAS	A5	A6	A4	A5 > A6 > A4 > A3 > A2 > A1
VIKOR	A5	A6	A4	A5 > A6 > A4 > A3 > A2 > A1

3.5.4. Final ranking and interpretation

As shown in Table 8, alternative A5 (PA: Cement = 85:15) achieved the highest final utility value (0.9664) and was ranked first, whereas A6 (82:18) and A4 (88:12) were ranked second and third, respectively. A3 (91:09) ranked fourth, followed by A2 (94:06) in fifth place and A1 (97:03) in sixth place. The ranked results reflect the overall observation that, while higher cement contents generally improve strength performance, the 85:15 mix (A5) offers the best compromise between early- and long-term strength gains, compaction characteristics, and moisture demand. Although A6 (82:18) achieved the highest later - age UCS, A5 achieved the highest overall utility because it provided the best balance across the weighted criteria. The Entropy–MARCOS framework successfully consolidates multiple performance indicators and consistently ranks A5 (85:15 PA: Cement) as the best in the order. This shows that MCDM techniques can provide a basis for appropriate geotechnical selection and help address trade-offs in stabilization design.

The ranking demonstrates that the proposed Entropy–MARCOS framework effectively converts multiple engineering performance indicators into a single, transparent decision metric, thereby facilitating the objective selection of the optimal stabilization mixture.

3.5.5. Implications

The findings of this study have several implications, as outlined below.

- **Engineering implication (mix selection for practice).** The Entropy–MARCOS framework converts the laboratory compaction–strength dataset into an auditable ranking of pond ash–cement mixes, enabling practitioners to select a mix based on multiple performance indicators rather than relying on a single-age UCS value or qualitative judgment. In the present dataset, the final ranking indicates A5 (PA: Cement = 85:15) as the best overall compromise solution (Final Utility = 0.9664), followed by A6 (82:18) and A4 (88:12).
- **Design/constructability implication (strength vs. moisture demand trade-off).** Entropy weighting assigns dominant importance to early-age UCS (7 days) (weight = 0.5369), followed by UCS at 14 and 28 days, while MDD and OMC contribute relatively less due to low dispersion across alternatives.
- **This means the ranking is driven primarily by strength development, but still penalizes alternatives that demand higher moisture (constructability sensitivity).** Practically, A5 performs best because it balances early strength and longer-term UCS while avoiding the highest moisture-demand penalty, even though A6 attains ideal/near-ideal values in some criteria.
- **Sustainability/decision-governance implication.** The study's contribution is not only the identification of an "optimal" PA–cement proportion for the tested materials, but also the demonstration of a replicable selection logic that can be transferred to other industrial by-products and stabilizer systems when multiple criteria (performance, constructability, durability surrogates, etc.) must be balanced.

An additional value of the proposed Entropy–MARCOS framework is its transferability to other engineering decision problems with minimal modification. The same structure can be applied wherever multiple alternatives must be evaluated against competing criteria. For example, in pavement material selection, alternatives may include cement-treated recycled aggregate, fly ash–stabilized soil, slag-based geopolymers mixtures, or lime-treated expansive soil, while criteria may include UCS, resilient modulus, California Bearing

Ratio (CBR), durability, cost, and environmental impact. By replacing the current decision matrix with the relevant engineering dataset, the Entropy method can objectively determine the criterion weights, and MARCOS can rank the alternatives based on their closeness to the ideal and anti-ideal solutions. Thus, the framework is not restricted to pond ash stabilization and can support broader material selection, infrastructure design, and sustainable construction decision-making.

The complete computational implementation and all intermediate calculations used to generate the reported results are available in the Appendix (Supplementary Material), enabling independent verification and reuse of the proposed framework.

4. Conclusions

This study investigated the stabilization of pond ash using varying cement contents to identify an optimal mix proportion that balances compaction behavior and strength development across multiple curing ages. Laboratory experiments were conducted to evaluate MDD, OMC, and UCS at 7, 14, and 28 days. To move beyond conventional single-criterion interpretation, the experimental results were integrated into a structured multi-criteria decision-making framework using Entropy-based objective weighting and the MARCOS ranking method. This approach enabled a transparent and quantitative comparison of alternative pond ash–cement mixes, accounting for trade-offs between early-age and long-term strength performance and constructability-related parameters.

The findings demonstrate that combining experimental testing with MCDM provides a robust decision-support tool for the rational selection of stabilized geomaterials, particularly in applications where industrial by-products are reused and multiple performance requirements must be satisfied simultaneously.

- **Objective weighting (Entropy) shows that strength dominates the decision.** Among the five criteria, UCS at 7 days received the highest entropy weight (0.5369), followed by UCS at 14 days (0.2201) and UCS at 28 days (0.2162), whereas MDD (0.0061) and OMC (0.0208) had minor influence due to limited variability.

- MARCOS ranking identifies the best overall PA–cement mix as A5 (85:15). Based on aggregated scores and utility degrees, A5 achieved the highest final utility (0.9664) and ranked first, while A6 ranked second and A4 ranked third.
- Best-compromise interpretation. Although A6 (82:18) can achieve very high strength performance, the Entropy–MARCOS synthesis indicates that A5 (85:15) offers the most balanced performance across early and later curing ages, together with compaction/moisture considerations, producing the best compromise solution under the adopted criteria and weights.
- Methodological contribution. The integrated Entropy–MARCOS workflow provides a transparent, data-driven pathway to convert experimental compaction/UCS datasets into a defensible mix-selection decision, suitable for pavement/geotechnical material design where multi-age strength and constructability must be considered simultaneously.
- Criteria scope limitation. The MCDM model employs MDD, OMC, and UCS at 7, 14, and 28 days. Durability indicators (wet–dry, freeze–thaw), stiffness/resilient modulus, shrinkage/cracking susceptibility, and environmental metrics (e.g., leaching) are not included; their inclusion could alter rankings.
- Local material specificity. Pond ash source and cement type are fixed (single PA source; OPC 53 grade). Results may vary with ash chemistry, fineness, Loss on Ignition (LOI), or different cementitious binders/activators.
- Weighting dependence on dataset dispersion. Entropy weights reflect dispersion in the dataset; therefore, if future datasets exhibit greater variability in OMC/MDD (or include additional criteria), the weights and ranking may shift.
- Uncertainty not explicitly modeled. Experimental variability (replicates, confidence intervals) is not propagated into the final utility values; rankings are deterministic.

Overall, the study demonstrates that cement-stabilized pond ash can be evaluated more rationally when experimental performance indicators are integrated with a structured decision-making framework. The Entropy–MARCOS approach enabled the simultaneous consideration of compaction behavior and strength development, leading to the selection of A5 (85:15 PA: Cement) as the most balanced mix. Beyond identifying an optimal proportion for the tested material system, the proposed framework provides a transparent, reproducible, and transferable approach for engineering material selection. The findings support the sustainable reuse of pond ash in geotechnical and pavement-related applications and establish a basis for future studies incorporating durability, environmental, economic, and field-performance criteria.

4.1. Limitations

Although the proposed Entropy–MARCOS intelligent decision framework provides a systematic and objective approach for optimizing cement-stabilized pond ash mixtures, several limitations should be recognized to properly interpret the present findings and guide future developments.

Recognizing these limitations provides a realistic interpretation of the present findings and establishes a clear foundation for extending the proposed framework to broader geotechnical optimization problems.

4.2. Future work

Building upon the findings of this study, several research directions can further enhance the applicability, robustness, and practical implementation of AI-assisted decision-support systems for sustainable geotechnical material optimization.

- Include other measure of performance – durability (wet-dry and freeze-thaw), shrinkage, stiffness modulus, indirect tensile strength (ITS) or flexural strength, and permeability to aid in the design of the pavement base and subbase.
- Environmental and sustainability coupling - include Leachability (Toxicity Characteristic Leaching Procedure, TCLP), binder embodied carbon and cost as additional criteria and then perform techno-enviro-economic Entropy-MARCOS ranking.
- Hybrid stabilization strategy: Considering combination of partial substituting fly ash, slag or geopolymer blends with cement and/or using cement

activators like lime and/or alkali activation and using the same AI-assisted MCDM pipeline for mix selection.

- Alternative weighting methods - Expand the basic approach to entropy based objective weighting to include other weighting schemes including Criteria Importance Through Intercriteria Correlation (CRITIC), Analytic Hierarchy Process (AHP), and Best-Worst Method (BWM). It may be beneficial to also investigate hybrid subjective-objective weighting methods, where experts' opinions (both geotechnical and pavement engineers) are also taken into account, and determine the influence of weighting philosophy on the overall final ranking results.
- Use of broader uncertainty analysis - Go beyond the deterministic sensitivity and comparative analyses of Section 3.5.3 and explicitly consider uncertainty in the decision model. These can take the form of Monte Carlo simulation of rank-probability distributions for the criteria values and weights, and/or fuzzy or grey-number based MCDM formulations that reflect the variability of the measurements and the scatter of the laboratories.
- Validation with other datasets - Test the transferability of the framework to other independent datasets that included other variations of soil types, binder systems, curing regimes and geographic regions etc., in order to assess the generalizability of the ranking methodology beyond the present dataset.
- Field validation - Build test sections or test embankment and subbase sections in the field with the highest ranked mixes (A5, A6 and A4), and assess the transferability from laboratory to field under seasonal moisture and traffic load.

In the end, the proposed AI-assisted Entropy-MARCOS framework offers a transparent, repeatable, and scalable computational decision support methodology that could speed up sustainable optimization of geomaterials and aide decision making aspects in geotechnical engineering applications based on data. Together, the research areas can help develop a more holistic, intelligent, and sustainable decision-support system for the future in geotechnical and pavement engineering.

Competing Interests Statement

The authors declare no conflict of interest.

Data Availability Statement

The experimental data supporting the findings of this study are included within the article. The complete Python source code used for Entropy weighting, MARCOS ranking, sensitivity analysis, comparative validation, and automated result generation is provided as Supplementary Material (Appendix). Additional data are available from the corresponding author upon reasonable request.

Declaration of Language Editing Tool Usage

The authors confirm that Grammarly Premium was used solely for English language editing, grammar correction, and readability improvement. The software was not used for content generation, data analysis, result interpretation, or scientific writing beyond linguistic refinement. All research design, experimental work, data processing, analysis, and manuscript preparation were conducted exclusively by the authors.

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Appendix

README for Appendix

Appendix provides the complete Python implementation used for the Entropy–MARCOS decision framework, sensitivity analysis, and comparative MCDM validation. The code was developed to ensure transparency, reproducibility, and independent verification of the ranking results reported in the manuscript.

The script was executed using Python 3.x. The required libraries are NumPy, Pandas, and OpenPyXL. NumPy is used for numerical calculations, Pandas is used for data handling and tabulation, and OpenPyXL is used to export results into Excel format.

The input data consists of the experimental decision matrix, which contains six PA–cement alternatives and five criteria: MDD, OMC, UCS at 7 days, UCS at 14 days, and UCS at 28 days. MDD and UCS criteria are treated as benefit criteria, whereas OMC is treated as a cost criterion.

The code should be executed sequentially from top to bottom. First, the decision matrix is defined. Second, Entropy weights are calculated. Third, MARCOS ranking is performed. Fourth, comparative validation is conducted using TOPSIS, COPRAS, EDAS, WASPAS, and VIKOR. Finally, sensitivity analysis is performed by varying the UCS 7-day weight by $\pm 10\%$ and $\pm 20\%$, as well as an equal-weight scenario.

The code automatically generates the Excel output file Entropy_MARCOS_Sensitivity_Comparative_Validation.xlsx, which contains the input matrix, the normalized matrix, the entropy weights, the MARCOS results, the comparative MCDM rankings, the method summary, and the sensitivity analysis. The tables reported in the manuscript can be reproduced directly from this Excel file.

This supplementary material supports complete computational reproducibility of the proposed decision-support framework.

```
import numpy as np
import pandas as pd

# -----
# Step 0: Input Decision Matrix
# -----
# Rows = PA:Cement mixes
# Columns = Criteria
criteria = ['MDD', 'OMC', 'UCS_7d', 'UCS_14d',
'UCS_28d']
alternatives = [
    '97:03', '94:06', '91:09',
    '88:12', '85:15', '82:18'
]

X = np.array([
    [1.135, 28.2, 0.524, 0.796, 0.970],
    [1.140, 28.0, 0.595, 0.970, 1.066],
    [1.193, 27.8, 0.696, 1.198, 1.338],
    [1.194, 27.0, 0.872, 1.254, 1.371],
    [1.196, 25.8, 1.185, 1.219, 1.587],
    [1.239, 24.0, 1.006, 1.425, 1.588]
])

df_X = pd.DataFrame(X, index=alternatives,
columns=criteria)
print("\nSTEP 1: Original Decision Matrix")
print(df_X.round(4))

# -----
# Step 1: Normalization
# -----
P = X / X.sum(axis=0)
df_P = pd.DataFrame(P, index=alternatives,
columns=criteria)

print("\nSTEP 2: Normalized Decision Matrix")
print(df_P.round(4))

# -----
# Step 2: Entropy Calculation
# -----
m, n = P.shape
k = 1 / np.log(m)

entropy = -k * np.sum(P * np.log(P + 1e-12), axis=0)
df_entropy = pd.DataFrame(entropy, index=criteria,
columns=['Entropy'])

print("\nSTEP 3: Entropy Values for Each Criterion")
print(df_entropy.round(4))

# -----
# Step 3: Degree of Diversification
# -----
diversification = 1 - entropy
df_div = pd.DataFrame(diversification, index=criteria,
columns=['Degree_of_Diversification'])
```

```

print("\nSTEP 4: Degree of Diversification")
print(df_div.round(4))

# -----
# Step 4: Entropy Weights
# -----
weights = diversification / diversification.sum()
df_weights = pd.DataFrame(weights, index=criteria,
columns=['Entropy_Weight'])

print("\nSTEP 5: Final Entropy Weights")
print(df_weights.round(4))

```

```

import numpy as np
import pandas as pd

# -----
# STEP 0: Input data
# -----
criteria = ['MDD', 'OMC', 'UCS_7d', 'UCS_14d',
'UCS_28d']
alternatives = ['97:03', '94:06', '91:09', '88:12', '85:15',
'82:18']

# Decision matrix (same as you used for entropy)
X = np.array([
[1.135, 28.2, 0.524, 0.796, 0.970],
[1.140, 28.0, 0.595, 0.970, 1.066],
[1.193, 27.8, 0.696, 1.198, 1.338],
[1.194, 27.0, 0.872, 1.254, 1.371],
[1.196, 25.8, 1.185, 1.219, 1.587],
[1.239, 24.0, 1.006, 1.425, 1.588]
])

df_X = pd.DataFrame(X, index=alternatives,
columns=criteria)
print("\nSTEP 1: Original Decision Matrix")
print(df_X.round(4))

# Entropy weights (your final)
w = np.array([0.0061, 0.0208, 0.5369, 0.2201, 0.2162])
df_w = pd.DataFrame({'Entropy_Weight': w},
index=criteria)
print("\nSTEP 2: Criteria Weights (Entropy)")
print(df_w.round(4))

# Criteria types: True=benefit, False=cost
benefit = np.array([True, False, True, True, True])

# -----
# STEP 1 (MARCOS): Determine AAI and AI
# AAI = Anti-ideal solution, AI = Ideal solution
# For benefit: AAI=min, AI=max
# For cost: AAI=max, AI=min
# -----
AAI = np.where(benefit, X.min(axis=0), X.max(axis=0))
AI = np.where(benefit, X.max(axis=0), X.min(axis=0))

```

```

df_AAI_AI = pd.DataFrame([AAI, AI], index=['AAI (Anti-
ideal)', 'AI (Ideal)'], columns=criteria)
print("\nSTEP 3: Anti-Ideal (AAI) and Ideal (AI)
Solutions")
print(df_AAI_AI.round(4))

# -----
# STEP 2 (MARCOS): Extended matrix
# (AAI row + alternatives + AI row)
# -----
X_ext = np.vstack([AAI, X, AI])
ext_index = ['AAI'] + alternatives + ['AI']
df_ext = pd.DataFrame(X_ext, index=ext_index,
columns=criteria)

print("\nSTEP 4: Extended Decision Matrix (AAI +
Alternatives + AI)")
print(df_ext.round(4))

# -----
# STEP 3 (MARCOS): Normalization
# Common MARCOS normalization:
# For benefit:  $n_{ij} = x_{ij} / x_{AIj}$ 
# For cost:  $n_{ij} = x_{AIj} / x_{ij}$ 
# (AI is ideal)
# -----
N = np.zeros_like(X_ext, dtype=float)
for j in range(len(criteria)):
    if benefit[j]:
        N[:, j] = X_ext[:, j] / AI[j]
    else:
        N[:, j] = AI[j] / X_ext[:, j]

df_N = pd.DataFrame(N, index=ext_index,
columns=criteria)
print("\nSTEP 5: Normalized Matrix (MARCOS)")
print(df_N.round(6))

# -----
# STEP 4 (MARCOS): Weighted normalized matrix
#  $v_{ij} = n_{ij} * w_j$ 
# -----
V = N * w
df_V = pd.DataFrame(V, index=ext_index,
columns=criteria)

print("\nSTEP 6: Weighted Normalized Matrix")
print(df_V.round(6))

# -----
# STEP 5 (MARCOS): Sum across criteria
#  $S_i = \sum_j v_{ij}$ 
# -----
S = V.sum(axis=1)
df_S = pd.DataFrame({'S_i': S}, index=ext_index)

print("\nSTEP 7: Aggregated Scores (S_i)")
print(df_S.round(6))

```

```

S_AAI = df_S.loc['AAI', 'S_i']
S_AI = df_S.loc['AI', 'S_i']

# -----
# STEP 6 (MARCOS): Utility degrees relative to AAI and AI
#  $K_i^- = S_i / S_{AAI}$ 
#  $K_i^+ = S_i / S_{AI}$ 
# -----
K_minus = S / S_AAI
K_plus = S / S_AI

df_K = pd.DataFrame({'K_minus (S_i/S_AAI)': K_minus,
                     'K_plus (S_i/S_AI)': K_plus}, index=ext_index)

print("\nSTEP 8: Utility Degrees Relative to AAI and AI")
print(df_K.round(6))

# -----
# STEP 7 (MARCOS): Final utility function
# A common crisp final utility:
#  $f(K_i) = (K_i^+ + K_i^-) / (1 + (S_{AI} / S_{AAI}))$ 
# (keeps consistency and ranking stability)
# -----
den = 1 + (S_AI / S_AAI)
F = (K_plus + K_minus) / den

df_F = pd.DataFrame({'Final_Utility_F': F},
                     index=ext_index)
print("\nSTEP 9: Final Utility Function (F)")
print(df_F.round(6))

# -----
# STEP 8: Ranking (exclude AAI and AI)
# -----
df_rank = df_F.loc[alternatives].copy()
df_rank['Rank'] =
df_rank['Final_Utility_F'].rank(ascending=False,
method='dense').astype(int)
df_rank = df_rank.sort_values('Final_Utility_F',
ascending=False)

print("\nSTEP 10: Final MARCOS Ranking (Higher F =
Better)")
print(df_rank.round(6))

```

```

import numpy as np
import pandas as pd

# -----
# 1. Input decision matrix
# -----
alternatives = ["A1", "A2", "A3", "A4", "A5", "A6"]
mixes = ["97:03", "94:06", "91:09", "88:12", "85:15",
"82:18"]

criteria = ["MDD", "OMC", "UCS_7d", "UCS_14d",
"UCS_28d"]
types = ["benefit", "cost", "benefit", "benefit", "benefit"]

X = pd.DataFrame({
    "MDD": [1.135, 1.140, 1.193, 1.194, 1.196, 1.239],
    "OMC": [28.2, 28.0, 27.8, 27.0, 25.8, 24.0],
    "UCS_7d": [0.524, 0.595, 0.696, 0.872, 1.185, 1.006],
    "UCS_14d": [0.796, 0.970, 1.198, 1.254, 1.219,
1.425],
    "UCS_28d": [0.970, 1.066, 1.338, 1.371, 1.587,
1.588]
}, index=alternatives)

# -----
# 2. Entropy weighting
# -----
P = X / X.sum(axis=0)
k = 1 / np.log(len(X))

entropy = -k * (P * np.log(P)).sum(axis=0)
diversification = 1 - entropy
weights = diversification / diversification.sum()

entropy_df = pd.DataFrame({
    "Criterion": criteria,
    "Type": types,
    "Entropy": entropy.values,
    "Diversification": diversification.values,
    "Entropy Weight": weights.values
})

# -----
# 3. MARCOS method
# AAI = Anti-ideal solution, AI = Ideal solution
# -----
def marcos_method(data, weights, types):
    ideal = []
    anti_ideal = []

    for col, t in zip(data.columns, types):
        if t == "benefit":
            ideal.append(data[col].max())
            anti_ideal.append(data[col].min())
        else:
            ideal.append(data[col].min())
            anti_ideal.append(data[col].max())

    extended = pd.DataFrame(

```

```

[anti_ideal] + data.values.tolist() + [ideal],
index=["AAI"] + list(data.index) + ["AI"],
columns=data.columns
)

norm = extended.copy()
for col, t, ideal_val in zip(data.columns, types, ideal):
    if t == "benefit":
        norm[col] = extended[col] / ideal_val
    else:
        norm[col] = ideal_val / extended[col]

weighted = norm * weights.values
S = weighted.sum(axis=1)

final_utility = S.loc[data.index] / S.loc["AI"]
ranks =
final_utility.rank(ascending=False).astype(int)

result = pd.DataFrame({
    "PA:Cement": mixes,
    "Aggregated Score": S.loc[data.index].values,
    "Utility Relative to AAI": (S.loc[data.index] /
S.loc["AAI"]).values,
    "Utility Relative to AI": final_utility.values,
    "Final Utility": final_utility.values,
    "Rank": ranks.values
}, index=data.index)

return extended, norm, weighted, result

extended, marcos_norm, marcos_weighted,
marcos_result = marcos_method(X, weights, types)

# -----
# 4. TOPSIS
# -----
def topsis(data, weights, types):
    norm = data / np.sqrt((data**2).sum(axis=0))
    weighted = norm * weights.values

    ideal, anti = [], []
    for col, t in zip(data.columns, types):
        if t == "benefit":
            ideal.append(weighted[col].max())
            anti.append(weighted[col].min())
        else:
            ideal.append(weighted[col].min())
            anti.append(weighted[col].max())

    ideal = np.array(ideal)
    anti = np.array(anti)

    d_pos = np.sqrt(((weighted - ideal) **
2).sum(axis=1))
    d_neg = np.sqrt(((weighted - anti) ** 2).sum(axis=1))
    score = d_neg / (d_pos + d_neg)

    return pd.Series(score, index=data.index)

```

```

# -----
# 5. COPRAS
# -----
def copras(data, weights, types):
    norm = data / data.sum(axis=0)
    weighted = norm * weights.values

    benefit_cols = [c for c, t in zip(data.columns, types) if
t == "benefit"]
    cost_cols = [c for c, t in zip(data.columns, types) if t
== "cost"]

    S_plus = weighted[benefit_cols].sum(axis=1)
    S_minus = weighted[cost_cols].sum(axis=1)

    Q = S_plus + (S_minus.min() * S_minus.sum()) /
(S_minus * ((S_minus.min() / S_minus).sum()))
    score = Q / Q.max()

    return score

# -----
# 6. EDAS
# -----
def edas(data, weights, types):
    avg = data.mean(axis=0)
    PDA = pd.DataFrame(index=data.index,
columns=data.columns, dtype=float)
    NDA = pd.DataFrame(index=data.index,
columns=data.columns, dtype=float)

    for col, t in zip(data.columns, types):
        if t == "benefit":
            PDA[col] = np.maximum(0, (data[col] -
avg[col]) / avg[col])
            NDA[col] = np.maximum(0, (avg[col] -
data[col]) / avg[col])
        else:
            PDA[col] = np.maximum(0, (avg[col] -
data[col]) / avg[col])
            NDA[col] = np.maximum(0, (data[col] -
avg[col]) / avg[col])

    SP = (PDA * weights.values).sum(axis=1)
    SN = (NDA * weights.values).sum(axis=1)

    NSP = SP / SP.max()
    NSN = 1 - SN / SN.max()

    score = (NSP + NSN) / 2
    return score

# -----
# 7. WASPAS
# -----
def waspas(data, weights, types, lam=0.5):
    R = data.copy()

```

```

for col, t in zip(data.columns, types):
    if t == "benefit":
        R[col] = data[col] / data[col].max()
    else:
        R[col] = data[col].min() / data[col]

SAW = (R * weights.values).sum(axis=1)
WPM = np.prod(R ** weights.values, axis=1)

score = lam * SAW + (1 - lam) * WPM
return pd.Series(score, index=data.index)

# -----
# 8. VIKOR
# -----
def vikor(data, weights, types, v=0.5):
    f_star = []
    f_minus = []

    for col, t in zip(data.columns, types):
        if t == "benefit":
            f_star.append(data[col].max())
            f_minus.append(data[col].min())
        else:
            f_star.append(data[col].min())
            f_minus.append(data[col].max())

    f_star = np.array(f_star)
    f_minus = np.array(f_minus)

    S, R = [], []

    for i in range(len(data)):
        terms = []
        for j, col in enumerate(data.columns):
            denom = abs(f_star[j] - f_minus[j])
            if denom == 0:
                term = 0
            elif types[j] == "benefit":
                term = weights.iloc[j] * (f_star[j] - data.iloc[i,
j]) / denom
            else:
                term = weights.iloc[j] * (data.iloc[i, j] -
f_star[j]) / denom
            terms.append(term)

        S.append(sum(terms))
        R.append(max(terms))

    S = np.array(S)
    R = np.array(R)

    Q = v * (S - S.min()) / (S.max() - S.min()) + \
        (1 - v) * (R - R.min()) / (R.max() - R.min())

    score = 1 - Q
    return pd.Series(score, index=data.index)

# -----

```

```

# 9. Comparative MCDM validation
# -----
method_scores = {
    "MARCOS": marcos_result["Final Utility"],
    "TOPSIS": topsis(X, weights, types),
    "COPRAS": copras(X, weights, types),
    "EDAS": edas(X, weights, types),
    "WASPAS": waspas(X, weights, types),
    "VIKOR": vikor(X, weights, types)
}

comparative_df = pd.DataFrame(index=alternatives)
comparative_df["PA:Cement"] = mixes

for method, score in method_scores.items():
    comparative_df[f"{method} Score"] = score
    comparative_df[f"{method} Rank"] =
score.rank(ascending=False).astype(int)

summary_rows = []
for method, score in method_scores.items():
    ranks = score.rank(ascending=False).astype(int)
    order = list(ranks.sort_values().index)
    summary_rows.append({
        "Method": method,
        "Rank 1": order[0],
        "Rank 2": order[1],
        "Rank 3": order[2],
        "Ranking Order": "> ".join(order)
    })

method_summary = pd.DataFrame(summary_rows)

# -----
# 10. Sensitivity analysis
# -----
def perturb_weights(base_weights, criterion, change):
    w = base_weights.copy()
    old = w[criterion]
    w[criterion] = old * (1 + change)

    diff = w[criterion] - old
    others = [c for c in w.index if c != criterion]

    for c in others:
        w[c] = base_weights[c] - diff * (base_weights[c] /
base_weights[others].sum())

    return w / w.sum()

sensitivity_rows = []

scenarios = {
    "Base Entropy": weights,
    "UCS_7d -20%": perturb_weights(weights,
"UCS_7d", -0.20),
    "UCS_7d -10%": perturb_weights(weights,
"UCS_7d", -0.10),

```

```

    "UCS_7d +10%": perturb_weights(weights,
    "UCS_7d", 0.10),
    "UCS_7d +20%": perturb_weights(weights,
    "UCS_7d", 0.20),
    "Equal Weights": pd.Series([0.2]*5, index=criteria)
    }

    for scenario, w in scenarios.items():
        _, _, result = marcos_method(X, w, types)

        row = {"Scenario": scenario}
        for c in criteria:
            row[f"{c} Weight"] = w[c]

        for alt in alternatives:
            row[f"{alt} Utility"] = result.loc[alt, "Final
Utility"]
            row[f"{alt} Rank"] = result.loc[alt, "Rank"]

        row["Best Alternative"] = result["Rank"].idxmin()
        sensitivity_rows.append(row)

    sensitivity_df = pd.DataFrame(sensitivity_rows)

    # -----
    # 11. Export all results to Excel
    # -----
    output_file =
    "Entropy_MARCOS_Sensitivity_Comparative_Validation.
xlsx"

```

```

    with pd.ExcelWriter(output_file, engine="openpyxl") as
    writer:
        input_df = X.copy()
        input_df.insert(0, "PA:Cement", mixes)
        input_df.insert(1, "Cement (%)", [3, 6, 9, 12, 15, 18])
        input_df.to_excel(writer, sheet_name="Input
Matrix")

        P.to_excel(writer, sheet_name="Entropy
Normalized")
        entropy_df.to_excel(writer, sheet_name="Entropy
Weights", index=False)

        extended.to_excel(writer, sheet_name="MARCOS
Extended Matrix")
        marcos_norm.to_excel(writer,
        sheet_name="MARCOS Normalized")
        marcos_weighted.to_excel(writer,
        sheet_name="MARCOS Weighted")
        marcos_result.to_excel(writer,
        sheet_name="MARCOS Results")

        comparative_df.to_excel(writer,
        sheet_name="Comparative MCDM")
        method_summary.to_excel(writer,
        sheet_name="Method Summary", index=False)
        sensitivity_df.to_excel(writer,
        sheet_name="Sensitivity Analysis", index=False)

        print("Excel file generated:", output_file)

```