

Effect of the Impact Model on the Dynamics of a Vibro-Impact System with Non-Ideal Excitation

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Abstract

The influence of the impact model on the dynamic behavior of a vibro-impact system with non-ideal excitation is analyzed in this paper. The system consists of a crank-slider mechanism and an oscillator, and its mathematical model is derived using Lagrange's equations of motion. Three soft impact models (Spring-Damper, Hertz, and Brake) and one hard impact model (Newton) are considered. The system response is analyzed numerically using frequency response diagrams, phase portraits, displacement-time histories, and basins of attraction. Three response regions are identified for all four models: non-impact, impact, and multiple solution regions. The multiple solution region corresponds to excitation-parameter values for which the system response depends on the initial conditions. For all models, the non-impact regime occurs for $\kappa = 0.30$ – 0.40 , while the impact regime occurs for $\kappa = 0.45$ – 1.55 . The largest differences between the models occur in the multiple solution region. Its upper boundary is $\kappa = 1.70$ for the Newton model, $\kappa = 1.65$ for the Spring-Damper model, and $\kappa = 1.85$ for the Hertz and Brake models. At lower values of the excitation control parameter, all four models predict similar dynamic responses, whereas noticeable differences emerge at higher values. These results demonstrate that impact-model selection becomes increasingly important at higher excitation levels for accurately predicting the system response and its sensitivity to initial conditions.

Keywords: nonlinear dynamics, numerical simulation, vibro-impact, non-ideal excitation, impact models, basins of attraction.

1. Introduction

Applications of vibro-impact systems, either with ideal or non-ideal excitation, can be found in various branches of engineering. Understanding their dynamics and impact behavior is of crucial importance for this very reason. Some of the application areas include hand-held pneumatic hammers, concrete drilling machines, vibrations in turbomachinery, tall buildings, bridges, automotive gearboxes, power transmission systems, and similar structures. Due to nonlinearity and discontinuity, vibro-impact systems are highly complex mechanical systems. The presence of friction, excitation, nonlinear elastic elements, etc. increases the system nonlinearity,

whereas discontinuities arise from impacts. The behavior of vibro-impact systems can be described with nonlinear differential equations that are often solved numerically due to their complexity. This nonlinearity can result in complex phenomena such as bifurcations and even chaotic behavior.

Excitation of a vibro-impact system can be classified as ideal and non-ideal excitation. For systems with ideal excitation, various effects are neglected, resulting in a simplified system representation. For this reason, analysis of systems with non-ideal excitation is of great importance due to their complexity. If the excitation is non-ideal, the system can influence the behavior of the

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excitation source, while for ideal excitation this effect is neglected. The presence of non-ideal excitation increases the system sensitivity to parameter variations, where small changes in parameters can lead to significant changes in the system behavior. Strong nonlinearity of these types of systems and their zones of instability contribute to the importance of studying and analyzing them.

The analysis and modelling of vibro-impact systems are presented in [1]–[5]. The application of vibro-impact theory to capsule systems is investigated in [1] where impacts can enable translational motion of the model. It is proven that with the right choice of parameters, stable periodic motion of the system can be achieved. In [2] high-frequency vibro-impact drilling was analyzed with respect to the static force, amplitude, and frequency of the applied excitation. The reduced model accurately predicted the optimal force and amplitude, while a detailed model was required to determine the optimal excitation frequency. In [3], the complexity of the system is increased by introducing various friction models. It was concluded that, for larger mass ratios, Coulomb model can be sufficiently applied. The influence of the various friction models on the dynamics of the vibro-impact system with ideal excitation is investigated in [4]. In [5], modelling of a vibro-impact penetration of self-exciting percussive-rotary drill bit is presented. The results show that the application of vibro-impact drilling reduces the required thrust force and it increases the rate of penetration due to the fragmentation of the material. The proposed model provides a basis for the design of more efficient drilling systems.

In studies [6]–[9], non-ideal excitation is analyzed, where, unlike ideal excitation, a relationship exists between the excitation source and the system. Direct comparison of ideal and non-ideal excitation on a specific model is presented in [6]. It is concluded that with ideal excitation the system can be described as more stable, and with non-ideal excitation additional nonlinearity occurs. Analysis of the influence of stall torque on a vibro-impact system with non-ideal excitation is presented in [7]. It is concluded that for small values of stall torque the system behaves as one with non-ideal excitation, while for higher values it behaves almost as one with ideal excitation. In [8], non-ideal excitation is combined with impact dampers for controlling chaos. The system consists of a cart driven

by a rotor with a bouncing particle that moves between walls that come from an impact damper. The results show that impact dampers can efficiently control chaos and stabilize the system. Duffing oscillator driven by a limited power supply is analyzed in [9]. It is shown that non-ideal excitation influences the occurrence of highly intertwined basins of attraction and depending on the initial conditions, system can converge to different attractors. It has been shown that the introduction of amplitude constraints can suppress the topology of attractors, destroy chaotic attractors and lead to changes in the structure of basin boundaries. Research carried out in [6]–[9] shows that non-ideal excitation introduces new effects in the dynamics of the system and understanding it is the key for controlling real vibro-impact systems.

In studies [10]–[13], various vibro-impact systems are analyzed, from hard and soft impact to elastic-plastic impact models. In [10] and [11], it is shown how the choice of an impact model affects the system response. The vibro-impact system with a moving base is considered in [10]. Additionally, a comparison between a hard impact model (Newton law of impact) and a soft impact model (Spring-Damper) was performed in that study. The models were compared under equivalent energy dissipation conditions and produced similar results only for high contact stiffness and low energy dissipation. At higher dissipation levels, the soft impact model predicted prolonged contact and significantly different responses. In [11], the effect of the contact model on stability and response of the system is analyzed. Various impact models, including hard-impact, Spring-Damper, Hertz, and elastic-plastic models, are compared. Results show that choosing the right impact model affects the stability of the results and that simplified models can lead to incorrect solutions. In comparison with [10], the study in [11] incorporates an impact model with realistic material characteristics, as opposed to idealized impact models. Hertzian non-linear contact, representing soft contact, is described in [12]. A one-degree-of-freedom (1-DOF) impact system is presented, and the results show subharmonic and chaotic motions with higher contact stiffness. An experimental study of the coefficient of restitution in elastic-plastic impact is presented in [13]. The results show that a constant value of the coefficient of restitution is only applicable to fully plastic impact models, while in the elastic-plastic impact models, it

varies depending on the velocity of the impacts and other factors.

In contrast to [10], where hard and soft impact models were compared for a system with a moving base, and [11], where different contact formulations were examined in a broader impact-vibration context, the present study focuses on a vibro-impact system with non-ideal excitation. All four models are compared under equivalent steady-state energy-dissipation conditions, allowing the influence of impact-model selection on response regions, penetration, plastic deformation, and sensitivity to initial conditions to be evaluated directly. Although previous studies have investigated different impact models and vibro-impact systems with non-ideal excitation, a systematic comparison of hard and soft impact models for a vibro-impact system with non-ideal excitation remains limited. The objective of this study is to investigate the influence of different impact models on the response of such a system. In particular, the study examines the occurrence of impact, non-impact, and multiple solution regimes, as well as sensitivity to initial conditions under identical energy dissipation conditions. The main contribution of this work is the systematic comparison of hard and soft impact models, highlighting their influence on the predicted dynamic response of vibro-impact systems with non-ideal excitation.

The vibro-impact system analyzed in this paper is driven by a non-ideal energy source, and for a large value of stall torque it behaves almost as a system with ideal excitation. Excitation of this system can cause the system to oscillate without impacts or behave as a vibro-impact system depending on the chosen parameters. Four different impact models were considered and compared: Newton impact model, Spring-Damper impact model, Hertz impact model and Brake model of impact. Newton impact model represents a hard impact, while the Spring-Damper and Hertz impact models are considered soft. The Hertz impact model considers the material of the base (wall) and the oscillator while the Spring-Damper impact model is considered ideal. The Brake impact model takes into account the plastic deformation of the base. Material hardening was not taken into consideration during the plastic deformation. To enable realistic comparison of the models, parameters were chosen to achieve the same energy dissipation during impacts in steady state. The stated influence of impact-model selection, excitation

parameters, and initial conditions is examined in the following sections through mathematical formulation, frequency response diagrams, phase portraits, displacement-time histories, and basins of attraction.

The study is divided into four different sections. In Section 2, physical and mathematical models of the system are presented and mathematical models for every impact model are derived. Results of the numerical analysis of the system with various impact models are presented in Section 3, while Section 4 presents the conclusions of the study.

2. Physical and Mathematical Modelling

The vibro-impact system analyzed in this paper consists of a crank-slider mechanism and a mass-spring system (oscillator) attached to it. As shown in Figure 1, the lengths of the rods OA and AB are l and $4l$ respectively. The mass of the rod OA is m_1 . The sliders B and C , with masses m_3 and m_4 , are connected via a spring with stiffness c . The undeformed length of the spring is l_0 . Friction between the oscillator and the contact surface is considered in the form of a viscous friction force. It is assumed that the driving torque M depends on the angular velocity of the eccentric drive, representing the non-ideal excitation, meaning that the system affects the excitation through the motion. Slider C can collide with a stop (base) in certain regimes, and four different impact models are considered in the presented analysis. Parameters of the impact models were chosen in order to achieve the same loss of kinetic energy in the steady-state, allowing a realistic comparison of the models.

The mathematical model is derived by applying the second-order Lagrange equations to the mechanical system with two degrees of freedom shown in Figure 1. The chosen generalized coordinates are the displacement of slider C , denoted by y , and the angle of rotation of rod OA , denoted by φ . The kinetic energy of the system includes the rotational motion of the crank and the translational motion of the slider, while the potential energy consists of the gravitational and elastic energy stored in the spring. The contribution of rod AB to the total kinetic energy is neglected as a simplifying modelling assumption, since its angular variation during the considered motion is relatively small. The same

assumption was adopted in [6]. The generalized forces include the excitation torque, damping and, depending on the impact model, the contact force. The mathematical model of the system for the chosen generalized coordinates is:

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial E_k}{\partial \dot{y}} \right) - \frac{\partial E_k}{\partial y} + \frac{\partial E_p}{\partial y} &= Q_y \\ \frac{d}{dt} \left(\frac{\partial E_k}{\partial \dot{\varphi}} \right) - \frac{\partial E_k}{\partial \varphi} + \frac{\partial E_p}{\partial \varphi} &= Q_\varphi \end{aligned} \quad (1)$$

Mathematical model of the system is:

$$\begin{aligned} l^2 \ddot{\varphi} \left(\frac{m_1}{3} + m_3 A \right) + \frac{m_3}{2} l^2 \dot{\varphi}^2 \frac{\partial A}{\partial \varphi} + c l \Delta l \cdot \frac{\partial \Delta l}{\partial \varphi} &= Q_\varphi \\ m_4 \ddot{y} + b \dot{y} + c \left[y - l \left(\cos \varphi + \sqrt{16 - \sin^2 \varphi} \right) - l_0 \right] &= m_4 g - F_{\text{contact}} \end{aligned} \quad (2)$$

where:

$$\begin{aligned} A &= \sin^2 \varphi + \frac{4 \sin \varphi \sin 2\varphi}{\sqrt{16 - \sin^2 \varphi}} + \frac{4 \sin^2 2\varphi}{16 - \sin^2 \varphi} \\ \Delta l &= y - l \left(\cos \varphi + \sqrt{16 - \sin^2 \varphi} \right) - l_0 \\ Q_\varphi &= \kappa M_o \left(1 - \frac{\dot{\varphi}}{\kappa \lambda_o} \right) - \frac{m_1 g l \sin \varphi}{2} \\ &\quad - m_3 g \left(\sin \varphi + \frac{2 \sin 2\varphi}{\sqrt{16 - \sin^2 \varphi}} \right) \end{aligned} \quad (3)$$

The main difference between the mathematical formulation of the Newton model and those of the other impact models is the presence of the contact force. For the Newton impact model, no continuous contact force is introduced, i.e., $F_{\text{contact}} = 0$ between impacts. The impact is modeled as an instantaneous velocity change determined by the coefficient of restitution. In the other three models, the impact is represented by a finite contact force. The detailed analysis of the contact force is presented in the following section.

Figure 1 (a) represents the system with the Newton impact model. The determination of the post-impact velocities is based on the coefficient of restitution. The oscillator can hit the base, as mentioned, and the

coordinate y_{stop} represents the position of the fixed barrier.

Figure 1 (b) represents the second impact model where the oscillator collides with the base that is supported by a spring with the stiffness c_s , and a damper with the damping coefficient b_d . This represents a soft impact model, in which kinetic energy loss and the change in relative velocity occur due to the force F_{contact} acting while the oscillator is in contact with the base.

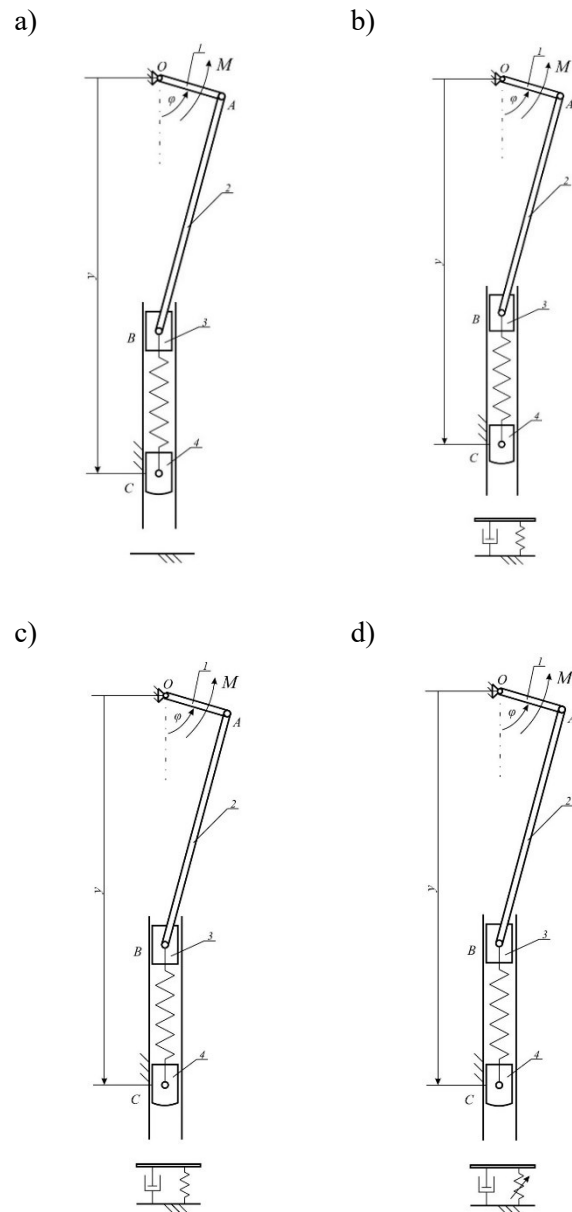


Figure 1. Impact models: (a) Newton impact model, (b) Spring-Damper impact model, (c) Hertz impact model, (d) Brake impact model (plastic regime).

The force $F_{contact}$ is defined as:

$$F_{contact} = c_s \delta + b_d \dot{y} \quad (4)$$

The parameter δ represents the indentation of the wall and it is defined as:

$$\delta = y - y_{stop} \quad (5)$$

The damper component of the force is active only when the relative velocity of the impact is positive, meaning when the oscillator is penetrating the base. Therefore, the force can be expressed as:

$$F_{contact} = c_s \delta + b_d \max(\dot{y}, 0) \quad (6)$$

Figure 1 (c) represents the system with the Hertz contact law. The stop base is also supported by a spring with the stiffness c_H defined as in [14]:

$$c_H = \frac{4}{3} E^* \sqrt{r} \quad (7)$$

and a damper with the damping coefficient:

$$b_H = \alpha c_H \quad (8)$$

defined by Hertz law. The parameter α is a damping parameter determined from the selected coefficient of restitution to obtain the same steady-state energy dissipation as in the other impact models. The Hertz contact law takes into consideration the materials of the colliding bodies. The base is modeled as aluminum with an elastic modulus of E_1 , and the Poisson's ratio of ν_1 , while the material of the slider C is modeled as steel with an elastic modulus of E_2 and Poisson's ratio of ν_2 .

The effective elastic modulus used to determine the Hertzian contact stiffness is defined as [14]:

$$E^* = \left(\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right)^{-1} \quad (9)$$

The effective radius is defined as in [14]:

$$r = \left(\frac{1}{r_1} + \frac{1}{r_2} \right)^{-1} \quad (10)$$

The radius of the base, r_1 , is chosen to represent a flat surface, while the radius of the collider is r_2 . In this case,

the Hertz law, which primarily describes impacts between two spheres, is applicable to the analyzed model. The loss of kinetic energy and the change in relative velocity are governed by the Hertzian contact force, defined as [12], [14]:

$$F_{contact} = c_H \delta^e + b_H \delta^e \dot{y} \quad (11)$$

The parameter e represents the Hertzian contact exponent and is equal to 3/2.

The parameter δ represents the penetration of the wall in the same way as previously described, [14]. The damping effect is only active when the oscillator is penetrating the base. Therefore, the force can be expressed as:

$$F_{contact} = c_H \delta^e + b_H \delta^e \max(\dot{y}, 0) \quad (12)$$

Figure 1(d) represents the system with a plastic regime, described by the Brake contact model. Effective modulus of elasticity as well as the effective radius used in this impact model, are the same as in the Hertzian contact. In addition to these parameters, the Brake impact model also considers yield strength, which in this analysis is assumed to correspond to the lower yield-strength value of the two materials considered:

$$\sigma_y = \min(\sigma_1, \sigma_2) \quad (13)$$

where σ_1 and σ_2 denote the yield strengths of the base and the slider materials, respectively.

According to the Brake contact model [11], the loading process is divided into elastic, mixed elastic-plastic, and fully plastic regimes, followed by an elastic restitution phase. The corresponding contact-force relations are summarized below.

a) Elastic regime

The contact force for the elastic regime is defined similarly to that in the Hertz impact model:

$$F_{contact} = \frac{4}{3} E^* \sqrt{r} \delta^{\frac{3}{2}} \quad (14)$$

for $0 \leq \delta \leq \delta_y$ and $0 \leq \dot{\delta}$. The penetration at the onset of yielding is defined as:

$$\delta_y = \frac{r}{f(v)} \left(\frac{\pi \sigma_y}{2E^*} \right)^2 \quad (15)$$

where $f(v)$ is the Poisson-ratio-dependent function defined in [11].

b) Mixed elastic-plastic regime

Contact force for $\delta_y \leq \delta \leq \delta_p$ and $0 \leq \delta$ [11]:

$$F_{contact}(\delta) = h_{00}(s)F_y + h_{10}(s)(\delta_p - \delta_y)F'_y + h_{01}(s)F_p + h_{11}(s)(\delta_p - \delta_y)F'_p \quad (16)$$

$$s = \frac{\delta - \delta_y}{\delta_p - \delta_y} \quad (17)$$

$$h_{00} = 2s^3 - 3s^2 + 1 \quad (18)$$

$$h_{10} = s^3 - 2s^2 + s \quad (19)$$

$$h_{01} = -2s^3 + 3s^2 \quad (20)$$

$$h_{11} = s^3 - s^2 \quad (21)$$

Here, F_y and F_p denote the contact-force values at δ_y and δ_p , respectively, while F'_y and F'_p are the corresponding derivatives with respect to penetration. The cubic Hermite interpolation ensures continuity of the contact force and its first derivative between the elastic and fully plastic regimes [11].

c) Fully plastic regime

For the fully plastic regime ($\delta > \delta_p$ and $0 \leq \delta$), the contact force is given by [11]:

$$F_{contact} = p_0 \pi (2r\delta + \xi) \quad (22)$$

Here, H is the effective Brinell hardness of the contacting materials and g is the gravitational acceleration used in the unit conversion to pascals [11]:

$$p_0 = Hg \cdot 10^6 \quad (23)$$

$$H = \left(\frac{2}{H_1} + \frac{2}{H_2} \right)^{-1} \quad (24)$$

$$\delta_p = \left(\frac{p_0}{\sigma_y} \right)^2 \delta_y \quad (25)$$

$$\xi = \left(\frac{3\pi p_0}{4E^*} r \right)^2 - 2 \left(\frac{p_0}{\sigma_y} \right)^2 \delta_p \quad (26)$$

d) Restitution phase

The assumption is that unloading is an elastic response without reverse yielding. The unloading phase begins after the oscillator reaches its maximum penetration and is not treated as a separate regime. If the maximum penetration does not exceed the yield displacement, the unloading response remains fully elastic.

The residual deformation of the base is defined as

$$\bar{\delta} = \delta_m - \left(\frac{3F_m}{4E^*\sqrt{\bar{r}}} \right)^{2/3} \quad (27)$$

where δ_m and F_m denote the maximum penetration and maximum contact force reached during loading, respectively.

For unloading following the fully plastic regime, the effective radius representing the deformed contact curvature is defined as:

$$\bar{r} = \frac{1}{2} \left(\delta_m + \frac{a_m^2}{\delta_m} \right) \quad (28)$$

where a_m denotes the maximum contact radius reached during loading.

For unloading from the mixed elastic-plastic regime, the effective radius is determined using cubic Hermite interpolation as given in [11]. For a fully elastic impact, $\bar{r} = r$ and $\bar{\delta} = 0$.

Contact force, for $\delta \leq 0$, is given by [11]:

$$F_{contact} = \frac{4}{3} E^* \sqrt{\bar{r}} (\delta - \bar{\delta})^{3/2} \quad (29)$$

Equation (29) applies while $\delta \geq \bar{\delta}$; after separation, $F_{contact} = 0$.

3. Numerical Analysis

Numerical analysis is conducted for the vibro-impact system using each impact model described in Section 2. In this study, numerical analysis is used to investigate the effect of the selected impact model on the response of the vibro-impact system with non-ideal excitation. The analysis enables the detection of different regimes of motion. Numerical analysis also investigates the system sensitivity to parameter variations. This leads to a better understanding of the system's behavior. To ensure a realistic comparison of the models, the simulations are performed under equivalent steady-state energy dissipation conditions.

For each representative case, quantitative values of the steady-state oscillation amplitude, maximum penetration depth, and permanent plastic deformation are reported alongside the corresponding phase portraits and displacement-time diagrams. These values are used to confirm and quantify the motion regimes previously identified from the frequency response diagrams and basins of attraction.

The representative phase portraits and displacement-time histories are included to compare how the four impact formulations describe collisions with equivalent steady-state kinetic-energy loss and to determine when the simpler Newton hard-impact model provides an adequate approximation of the soft-impact response.

3.1. Newton impact model

Figure 2 presents the frequency response diagram of the vibro-impact system for the Newton impact model. Based on the frequency response diagram, it can be concluded that steady-state displacement amplitude depends on the value of the excitation control parameter.

The horizontal axis represents the excitation control parameter (κ), and the simulation is performed over the range 0.3–2 with an increment of 0.05. The vertical axis represents the minimum and maximum (y_{min}, y_{max}) displacement of the oscillator C in the steady state motion for each considered value of the excitation control parameter. As mentioned before, this impact model represents a hard impact, and the base is impenetrable (rigid).

In Figure 2, it is shown that for certain values of the excitation control parameter, the system behaves as a vibro-impact system and for others it operates in a non-impact regime. For $0.3 \leq \kappa \leq 0.4$ or $1.75 \leq \kappa \leq 2$ regime of the system is non-impact, while for the values from $0.45 \leq \kappa \leq 1.55$ the impact regime is present. For a narrow range of values of the excitation control parameter (1.6-1.7), multiple solutions can be observed on the frequency response diagram. In this case, for the same value of parameter κ both the impact and non-impact regimes can be present. In this region, behavior of the system depends on the combination of initial conditions. Green lines shown in the diagram represent the minimum and maximum possible displacement of the oscillator.

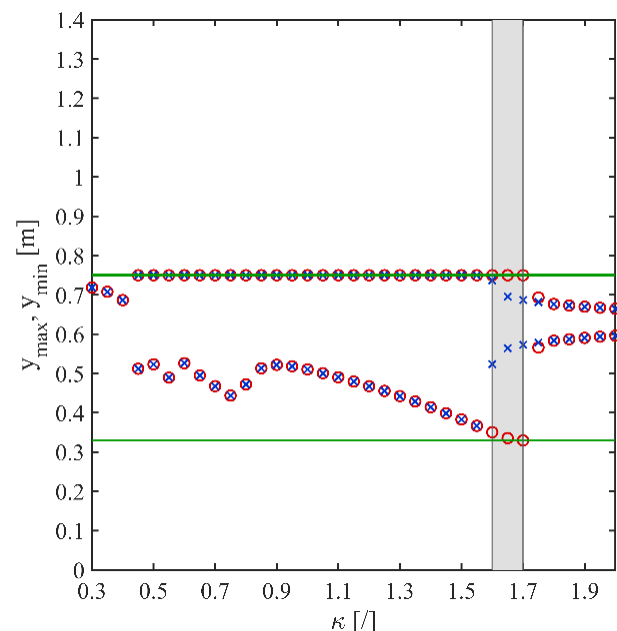


Figure 2. Frequency Response diagram - Newton impact model.

For the considered vibro-impact system, based on the frequency response diagram, two different excitation control parameters are chosen, corresponding to the impact and non-impact regions. For each of these excitation control parameters, phase portraits and displacement-time diagrams at steady-state motion are formed to show different regimes of the system. The simulation was performed for 200 seconds of motion.

Figure 3 represents the phase portrait of the system at steady-state motion for the value of the excitation control parameter of 1.8, confirming the non-impact dynamic behavior. The absence of impacts results in a closed and continuous phase trajectory. Figure 4 shows displacement

of the oscillator as a function of time over the selected interval from 190 to 200 seconds. The diagram confirms the non-impact behavior, while the amplitude of oscillations is approximately 0.0465 m.

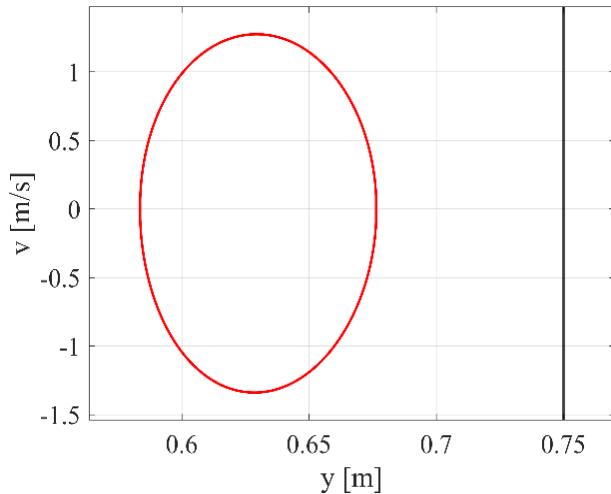


Figure 3. Phase portrait of the system for $\kappa = 1.8$.

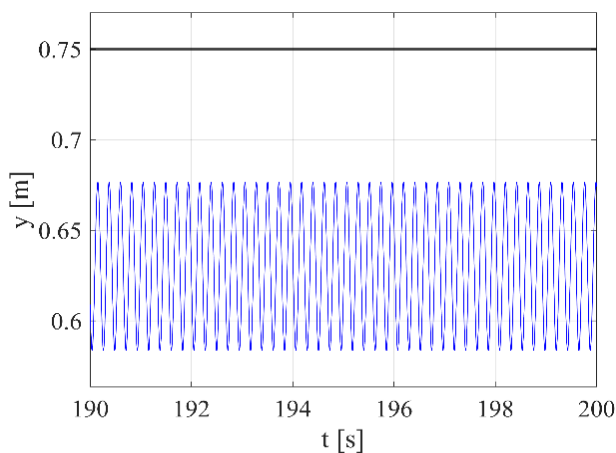


Figure 4. Displacement-time diagram for $\kappa = 1.8$.

The second response of the system, corresponding to the excitation control parameter value of 1.35, is presented in the following phase portrait and displacement-time diagram (Figure 5 and Figure 6). In this case, the system behaves as a vibro-impact system. The phase portrait shows the discontinuities in velocity at the moments of impact, which represent energy dissipation due to impacts. The displacement-time diagram also confirms the impact regime of motion. The amplitude of oscillations during the steady-state motion is approximately 0.1607 m.

In order to analyze the influence of the combinations of initial conditions on the regime of the system, a basin of attraction was generated for a specific value of the

excitation control parameter from the multiple solution region observed on the frequency response diagram. Blue dots shown in the diagram represent the combinations of initial conditions that lead to the non-impact regime, while red dots correspond to the impact regime of motion.

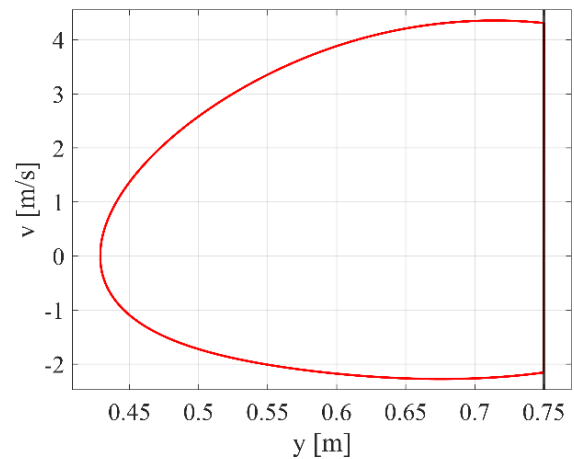


Figure 5. Phase portrait of the system for $\kappa = 1.35$.

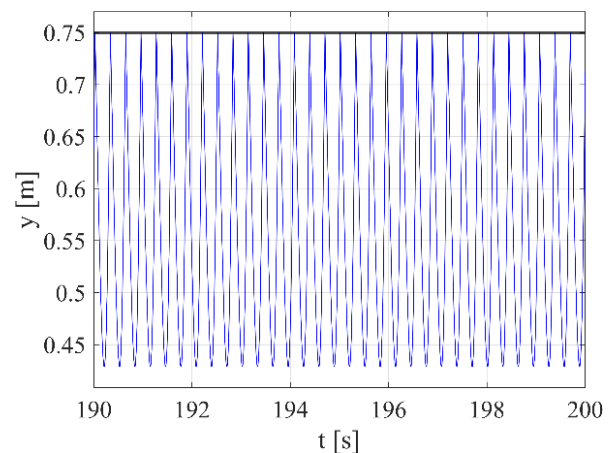


Figure 6. Displacement-time diagram for $\kappa = 1.35$.

Analysis of the basins of attraction is crucial for determining the sensitivity of the system to the initial conditions and for clearly identifying the impact and non-impact regions.

Figure 7 represents a basin of attraction at $\kappa=1.7$ for the system with the Newton impact model. The non-impact behavior is concentrated in the center of the diagram for smaller values of initial velocity, while for the greater values, regardless of the direction, the system always operates in the impact regime of motion. The boundaries of the non-impact region are not symmetrical about the horizontal axis and this structure of the basin of attraction emphasizes the importance of initial conditions for determining the behavior of the system.

For two combinations of initial conditions, one from a non-impact region and one from an impact region of the basin of attraction, phase portraits and displacement-time diagrams at steady-state motion were generated.

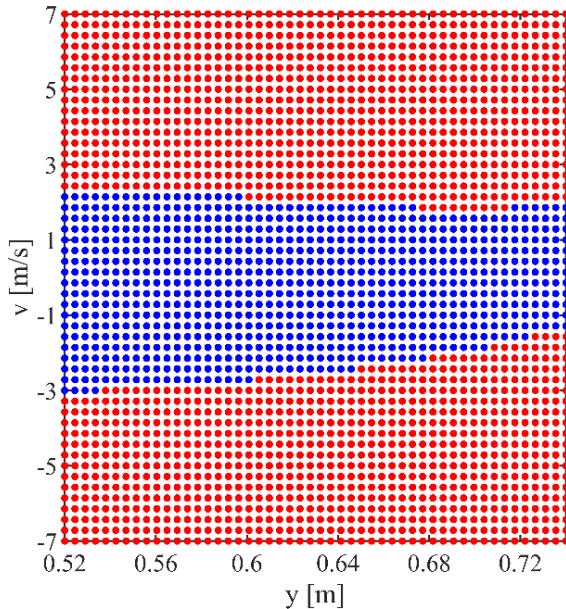


Figure 7. Basin of attraction diagram for $\kappa = 1.7$.

Figure 8 represents a phase portrait of the system for the initial displacement of 0.6 m and zero value of initial velocity. The phase trajectory forms a closed circle shaped loop, and the system does not establish the contact with the base during the motion. Figure 9 represents a displacement-time diagram for the same initial conditions as in the phase portrait shown in Figure 8 and confirms that the non-impact regime is present. The amplitude of the oscillations at steady-state motion is approximately 0.0557 m.

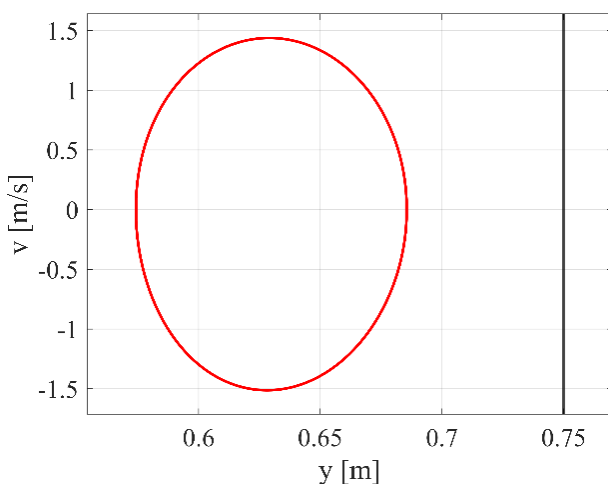


Figure 8. Phase portrait of the system for $\kappa = 1.7$, $y = 0.6$ m, $v = 0$ m/s.

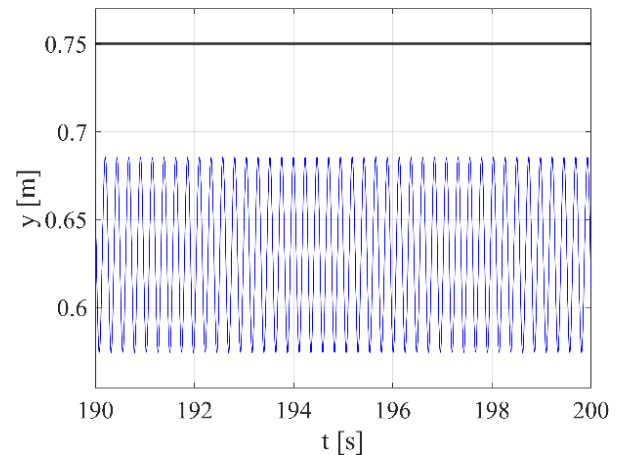


Figure 9. Displacement-time diagram for $\kappa = 1.7$, $y = 0.6$ m, $v = 0$ m/s.

In Figure 10, the phase portrait of the system for initial velocity of 3 m/s and the initial position of 0.6 m is represented. Figure 11 presents the displacement-time diagram for the same initial conditions as the phase portrait in Figure 10.

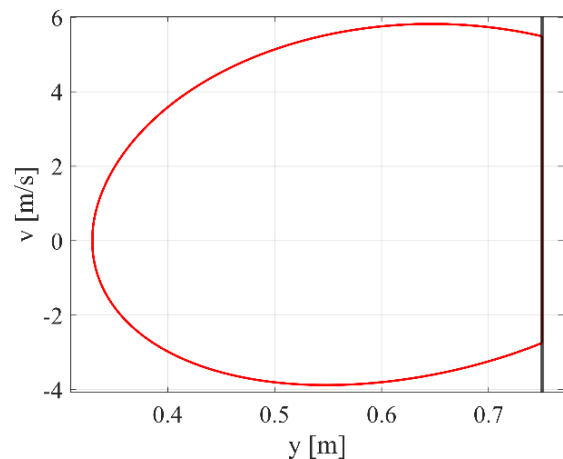


Figure 10. Phase portrait of the system for $\kappa = 1.7$, $y = 0.6$ m, $v = 3$ m/s.

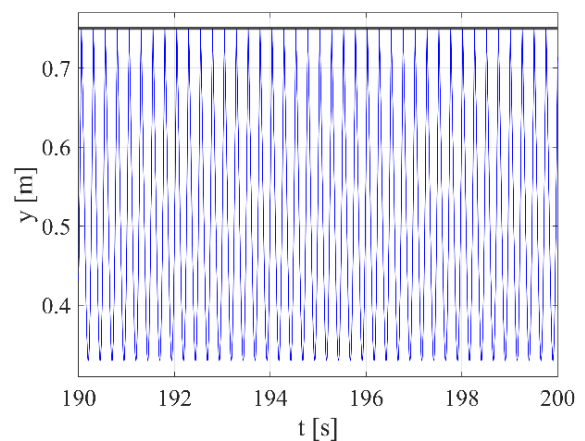


Figure 11. Displacement-time diagram for $\kappa = 1.7$, $y = 0.6$ m, $v = 3$ m/s.

The diagrams shown confirm the impact regime of the vibro-impact system for the combination of initial conditions from the red region of the basin of attraction. Based on the displacement-time diagram, the amplitude of the oscillations is approximately 0.2103 m.

3.2. Spring-Damper impact model

Figure 12 represents a frequency response diagram for the Spring-Damper impact model, and it is based on a similar procedure to that used for the Newton impact model. The only difference is that the Spring-Damper model represents the soft impact model, therefore the upper green line on the diagram represents the position of the base (wall) but it can be penetrated during the motion. Therefore, the base is not fixed as it was in the first impact model.

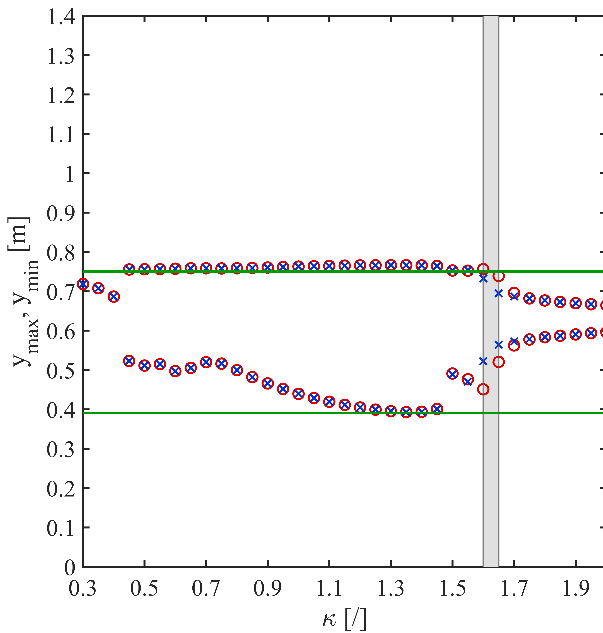


Figure 12. Frequency Response diagram - Spring-Damper impact model.

Based on the frequency response diagram shown in Figure 12, the impact and non-impact regions are present for the Spring-Damper impact model, as was the case for the system with the Newton impact model. Values of the excitation control parameter from $0.3 \leq \kappa \leq 0.4$ or $1.7 \leq \kappa \leq 2$ correspond to the non-impact regime. The impact regime of the system occurs for the interval $0.45 \leq \kappa \leq 1.55$. The multiple solution region is observed for the interval $1.6 \leq \kappa \leq 1.65$ and within this interval the regime depends on the initial conditions. Also, it can be

observed that an increase in the value of the excitation control parameter to 1.45 leads to an increase in wall penetration.

For $\kappa=1.75$, selected from the non-impact region of the frequency response diagram, the steady-state phase portrait is presented in Figure 13. The phase portrait confirms the non-impact response, since the maximum value of the oscillator displacement is approximately 0.6807 m. The oscillation amplitude at the steady-state motion is approximately equal to 0.0507 m.

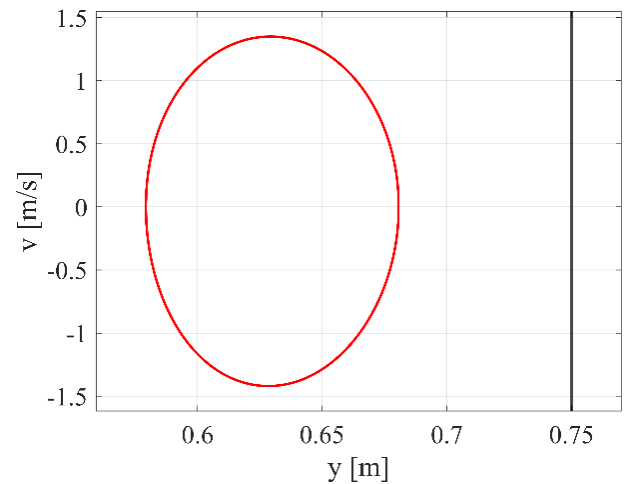


Figure 13. Phase portrait of the system for $\kappa = 1.75$.

The displacement-time diagram (Figure 14) shows that after approximately 35–40 seconds, the system reaches a steady state with an almost constant oscillation amplitude. The diagram also shows that the system operates in a strict non-impact regime which is confirmed by the phase portrait presented in Figure 13.

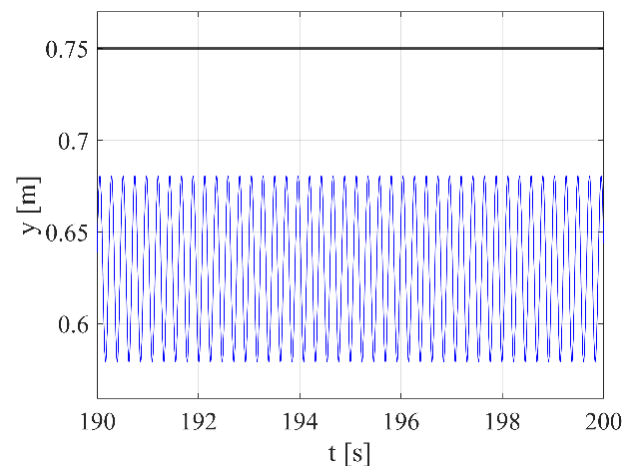


Figure 14. Displacement-time diagram for $\kappa = 1.75$.

The phase portrait is presented in Figure 15 for the value of the excitation control parameter of 1.35, and it confirms the impact behavior of the system. The vertical grey line on the diagram represents the initial position of the base which can be penetrated due to the nature of the soft impact model. The difference between the velocities before and after the impact indicates the dissipation of energy.

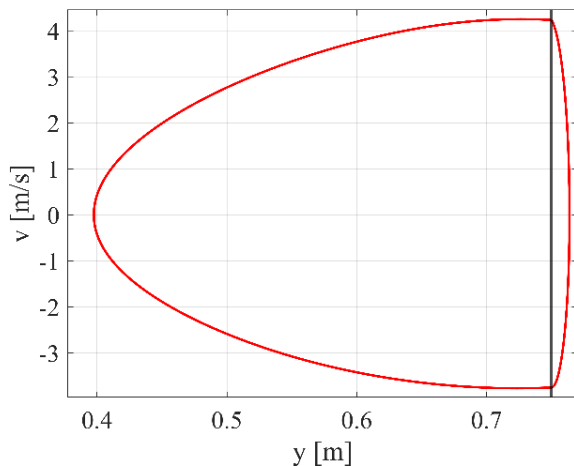


Figure 15. Phase portrait of the system for $\kappa = 1.35$.

The displacement-time diagram, presented in Figure 16, shows the oscillatory impact regime of the system. The oscillation amplitude is approximately equal to 0.1833 m during the steady state of motion. For the considered set of parameters, the maximum penetration depth is approximately 0.0139 m.

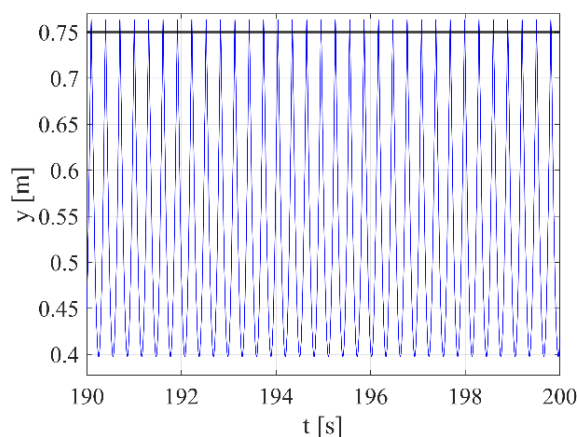


Figure 16. Displacement-time diagram for $\kappa = 1.35$.

Figure 17 represents the basin of attraction for the value of the excitation control parameter of 1.65, chosen from the multiple solution region observed on the frequency response diagram. The non-impact region is mostly concentrated around the smaller values of initial

velocity and initial displacement. The boundaries between the two mentioned regimes of motion are asymmetrical, indicating the high nonlinearity of the system as well as the strong dependence on the initial conditions.

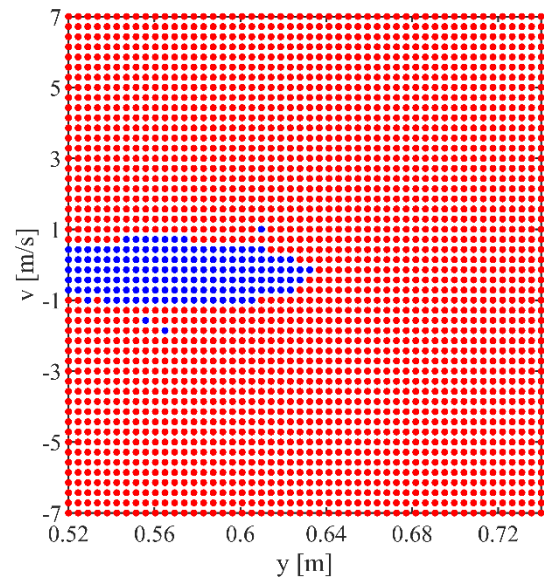


Figure 17. Basin of attraction diagram for $\kappa = 1.65$.

Figure 18 represents the phase portrait of the system for the selected initial conditions corresponding to the non-impact behavior. The initial position is 0.6 m, while the initial velocity is 0 m/s.

Figure 19 represents the displacement-time diagram for the same initial conditions. The system operates in a non-impact regime of motion during the steady-state motion. The oscillation amplitude is approximately 0.0619 m. The simulation was performed over a time interval of 900 seconds, as the previously considered interval was not sufficient for the system to reach steady-state motion.

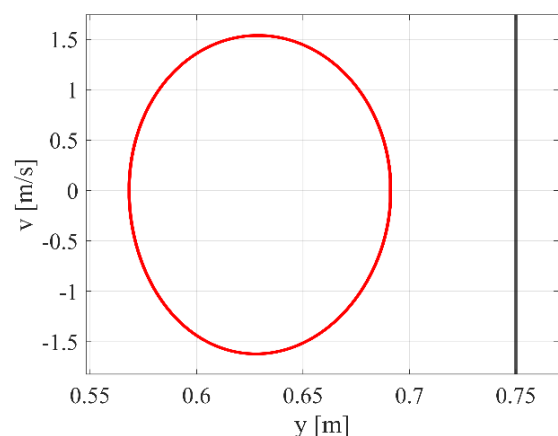


Figure 18. Phase portrait of the system for $\kappa = 1.65$, $y = 0.6$ m, $v = 0$ m/s.

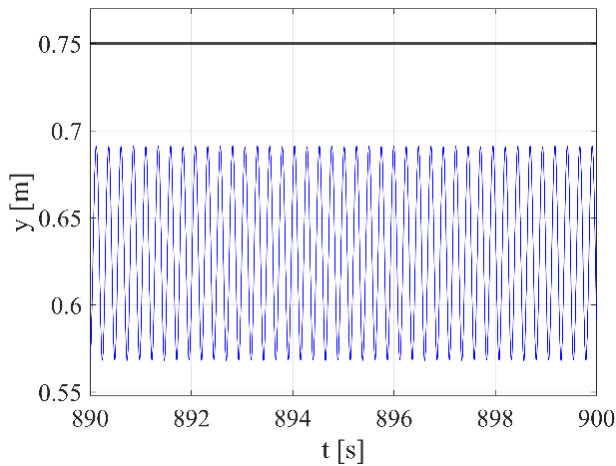


Figure 19. Displacement-time diagram for $\kappa = 1.65$, $y = 0.6$ m, $v = 0$ m/s.

In Figure 20, the phase portrait of the system at steady-state motion is shown for values of initial displacement of 0.6 m and initial velocity of 1 m/s. In this phase portrait, the impacts are clearly visible, as well as the penetration of the wall due to the soft impact. The maximum penetration depth is approximately 0.0258 m based on the phase portrait.

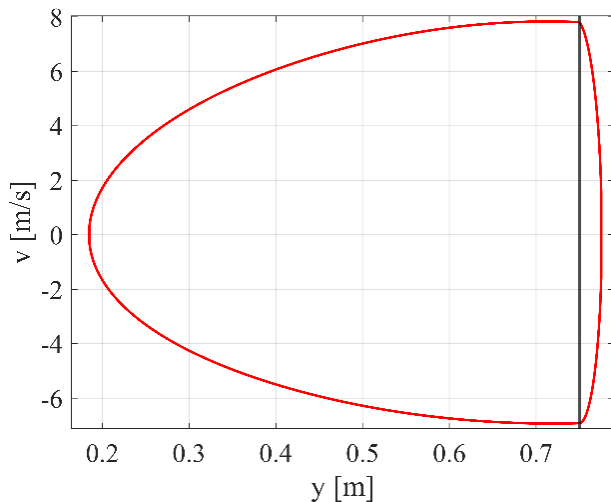


Figure 20. Phase portrait of the system for $\kappa = 1.65$, $y = 0.6$ m, $v = 1$ m/s.

The displacement-time diagram is presented in Figure 21 for the same combination of initial conditions as the phase portrait. The diagram confirms the impact regime, and the oscillation amplitude is approximately 0.2956 m.

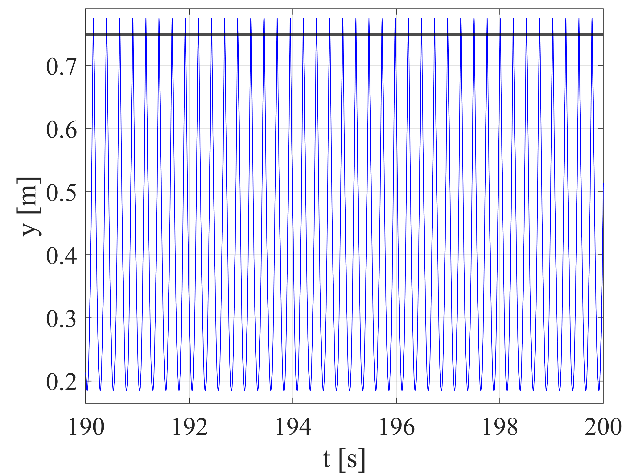


Figure 21. Displacement-time diagram for $\kappa = 1.65$, $y = 0.6$ m, $v = 1$ m/s.

3.3. Hertz impact model

Figure 22 represents a frequency response diagram for the Hertz impact model, and it is based on the same procedure as that used for the Spring-Damper impact model. The Hertz impact model also represents the soft impact model. Therefore, the upper green line on the diagram represents the position of the base (wall) but it can be penetrated during the motion.

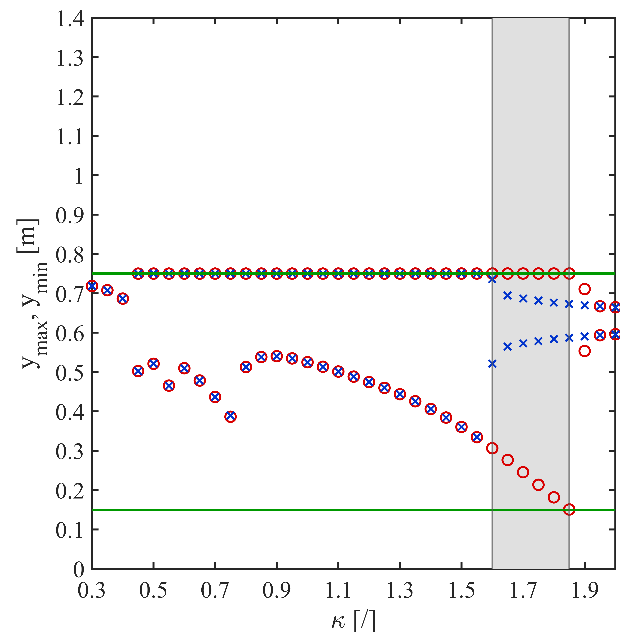


Figure 22. Frequency Response diagram - Hertz impact model.

Three distinct regions can be identified in the frequency response diagram for the Hertz impact model (Figure 22), as in the other two impact models. For the

excitation control parameter from $0.3 \leq \kappa \leq 0.4$ or $1.9 \leq \kappa \leq 2$ the regime of the system is non-impact, while the values from $0.45 \leq \kappa \leq 1.55$ correspond to the impact regime, regardless of initial conditions. The regime of the system depends on the combination of initial conditions for the excitation control parameter from $1.6 \leq \kappa \leq 1.85$. In this interval, multiple solutions occur, and the regime can be impact or non-impact.

The phase portrait shown in Figure 23, for $\kappa=1.95$, represents a non-impact regime of motion of the system, since the maximum displacement of the oscillator is approximately equal to 0.667 m.

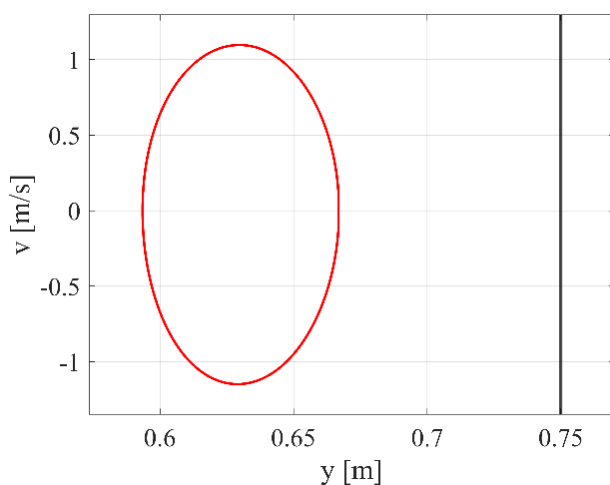


Figure 23. Phase portrait of the system for $\kappa = 1.95$.

Figure 24 represents a displacement-time diagram for the same value of the excitation control parameter, 1.95. The value of the oscillation amplitude at steady-state motion is approximately 0.0368 m.

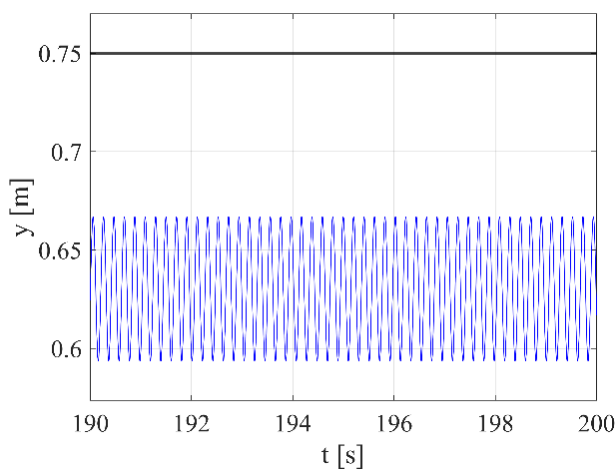


Figure 24. Displacement-time diagram for $\kappa = 1.95$.

Figure 25 represents the phase portrait of the system for the value of the excitation control parameter 1.35 and

it confirms the impact regime of motion observed in the frequency response diagram. Based on the phase portrait, the maximum penetration is approximately equal to 0.0002 m.

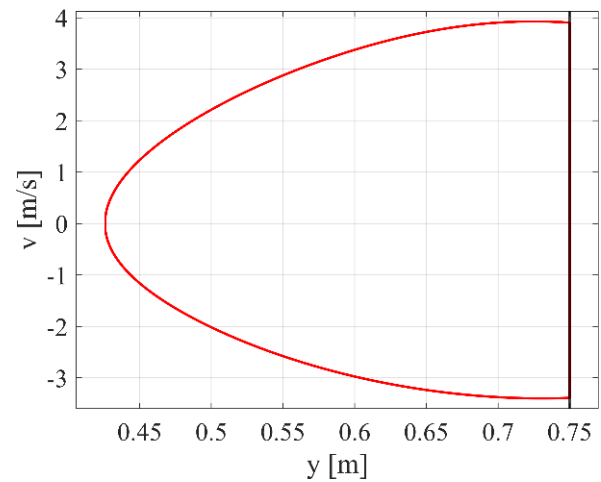


Figure 25. Phase portrait of the system for $\kappa = 1.35$.

The displacement-time diagram for the same value of excitation control parameter, $\kappa=1.35$, clearly illustrates the impact regime of motion (Figure 26). The amplitude of oscillations is approximately 0.1622 m.

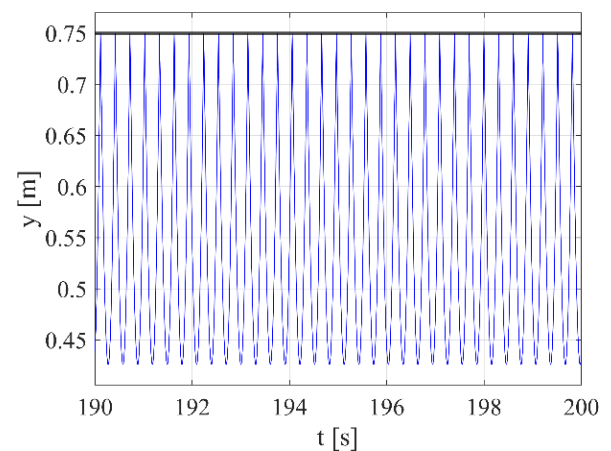


Figure 26. Displacement-time diagram for $\kappa = 1.35$.

For the vibro-impact system with the Hertz impact model, a basin of attraction for the excitation control parameter of 1.65 is presented in Figure 27. The value of the excitation control parameter is chosen from the multiple solution region observed in the frequency response diagram. The non-impact region is located around the smaller values of initial position and initial velocity. The boundaries between the two regions are asymmetrical around the horizontal axis. The basin of attraction shows that the impact regime occupies a larger portion of the basin.

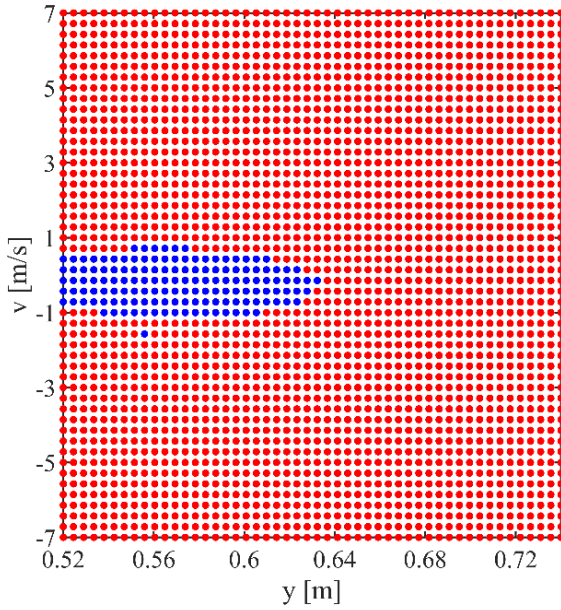


Figure 27. Basin of attraction diagram for $\kappa = 1.65$.

Phase portrait for the combination of initial conditions from the non-impact region is shown in Figure 28. The initial position is equal to 0.6 m, and the initial velocity is 0 m/s. The phase portrait confirms the non-impact regime of the system, since the oscillator does not collide with the barrier, represented by the grey line. The maximum displacement of the oscillator at steady-state motion is approximately 0.6917 m. Figure 29 represents the displacement-time diagram for the same initial conditions. The oscillator does not collide with the base during the motion for the chosen time interval. The amplitude of the oscillations is approximately 0.0619 m. The simulation was performed over 800 seconds to allow the system to reach steady-state motion for the chosen combination of parameters and initial conditions.

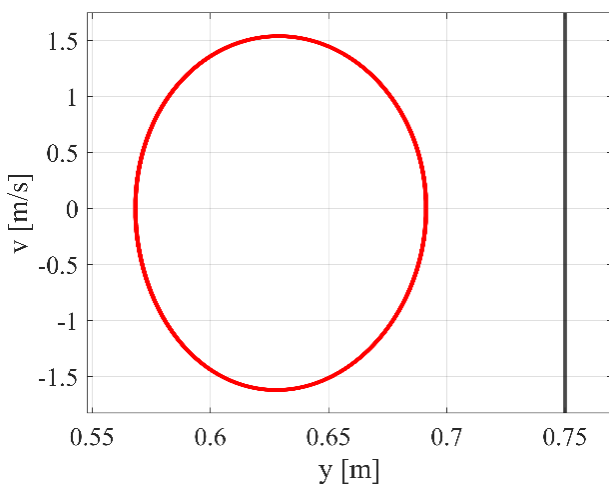


Figure 28. Phase portrait of the system for $\kappa = 1.65$, $y = 0.6$ m, $v = 0$ m/s.

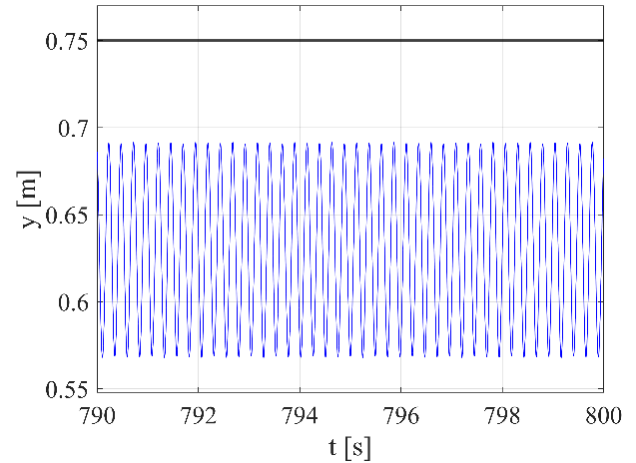


Figure 29. Displacement-time diagram for $\kappa = 1.65$, $y = 0.6$ m, $v = 0$ m/s.

For the combination of initial conditions corresponding to the red region of the basin of attraction, the phase portrait and displacement-time diagrams at steady-state motion are presented in Figures 30 and 31. The initial position is set to 0.6 m, and the initial velocity is 1 m/s. The Hertz soft impact model allows penetration into the wall due to the compliant nature of the contact model.

Since the Hertz impact model accounts for material properties, the penetration depth for the selected materials remains relatively small (approximately 0.0003 m). Energy dissipation during impacts is evident from the difference in velocity values immediately before and after the impact. The amplitude of oscillations is approximately equal to 0.2368 m.

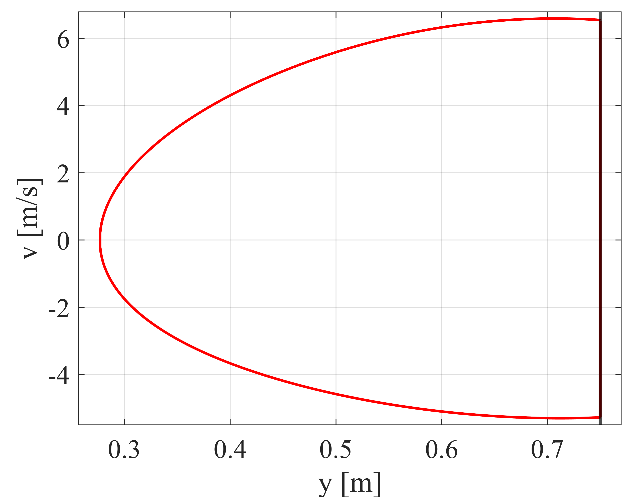


Figure 30. Phase portrait of the system for $\kappa = 1.65$, $y = 0.6$ m, $v = 1$ m/s.

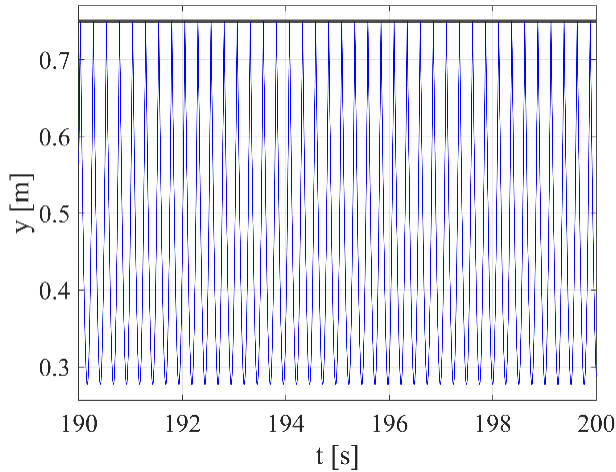


Figure 31. Displacement-time diagram for $\kappa = 1.65$, $y = 0.6$ m, $v = 1$ m/s.

3.4. Brake impact model

The frequency response diagram shown in Figure 32 corresponds to the Brake impact model and it is based on the same procedure used for the other soft impact models. The Brake model is also a soft impact formulation, but unlike the other models applied in this study, it includes elastic, elastic-plastic, and fully plastic contact regimes. Therefore, the green line on the diagram represents the position of the base (wall), which is not fixed. The position of the wall changes during the motion due to the plastic deformation. The frequency response diagram shows impact and non-impact regions, as in the other impact models. The values of excitation control parameter that correspond to the non-impact region are $0.3 \leq \kappa \leq 0.4$ or $1.9 \leq \kappa \leq 2$. Values from $0.45 \leq \kappa \leq 1.55$ correspond to the impact regime. The multiple solution region is present for values of the excitation control parameter $1.6 \leq \kappa \leq 1.85$.

Figure 33 represents the phase portrait of the system at steady-state motion for $\kappa=1.9$. The phase portrait confirms the non-impact region, and the maximum displacement is approximately 0.6698 m.

Figure 34 shows the displacement-time diagram for the same combination of initial conditions as the phase portrait shown in Figure 33. The amplitude of the oscillations is approximately 0.0396 m. During the motion, the maximum displacement does not exceed the value of 0.75 m that represents the position of the base. Therefore, this diagram confirms the non-impact regime of motion.

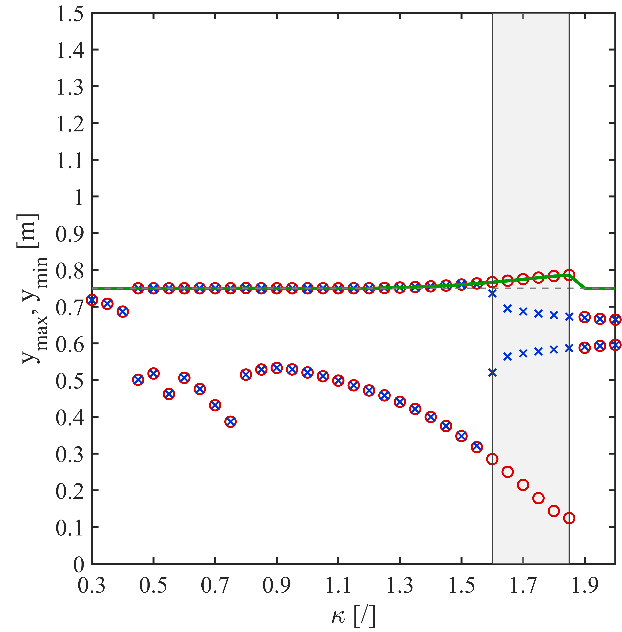


Figure 32. Frequency Response diagram - Brake contact model.

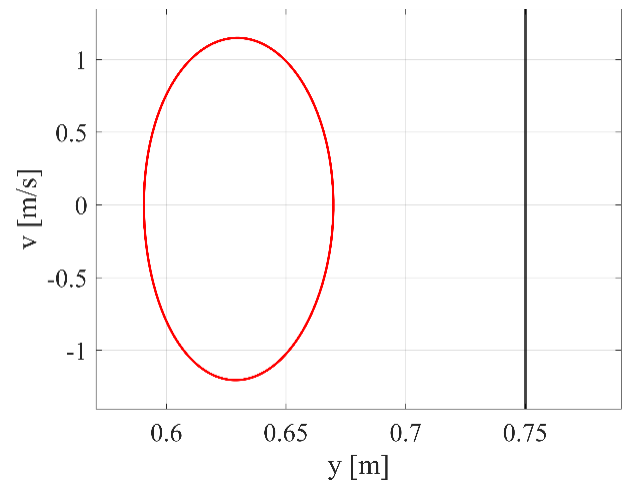


Figure 33. Phase portrait of the system for $\kappa = 1.9$.

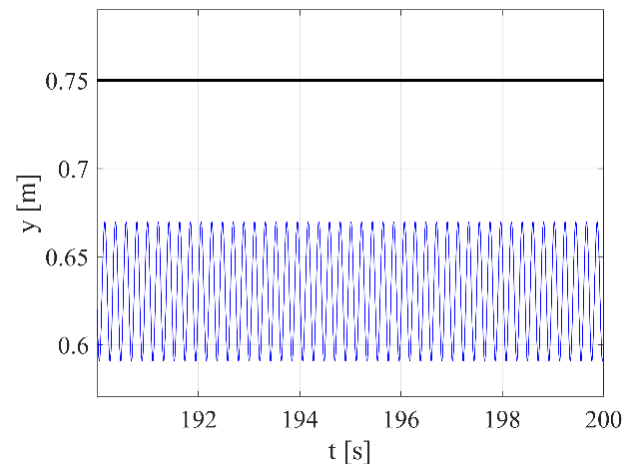


Figure 34. Displacement-time diagram for $\kappa = 1.9$.

The phase portrait (Figure 35) of the system for the value of the excitation control parameter of 1.35 displays the impact regime of motion and it confirms the behavior observed on the frequency response diagram.

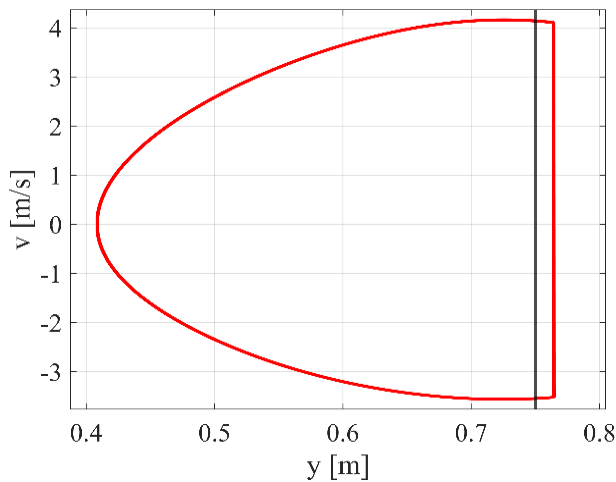


Figure 35. Phase portrait of the system for $\kappa = 1.35$.

The displacement-time diagram for the same value of excitation control parameter is presented in Figure 36. The impact regime can be observed and the average maximum penetration during impacts is approximately 0.000267 m. The average permanent plastic deformation is approximately 0.000023 m. The plastic deformation is considerably smaller than the total penetration. Therefore, the response is predominantly elastic, with a smaller irreversible deformation component.

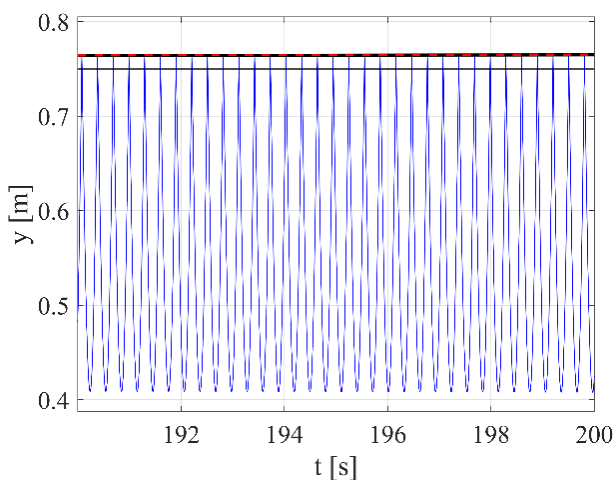


Figure 36. Displacement-time diagram for $\kappa = 1.35$.

A basin of attraction for $\kappa=1.7$ is presented in Figure 37. The chosen value of the excitation control parameter corresponds to the multiple solution region observed on the frequency response diagram. The dominant regime of

motion is the impact regime, represented by red dots. The non-impact region, represented by blue dots, is localized around the smaller values of the initial position and initial velocity. The influence of the initial position is more significant than the influence of the initial velocity, since for initial displacements greater than 0.66 m, the impact regime is present, regardless of the initial velocity.

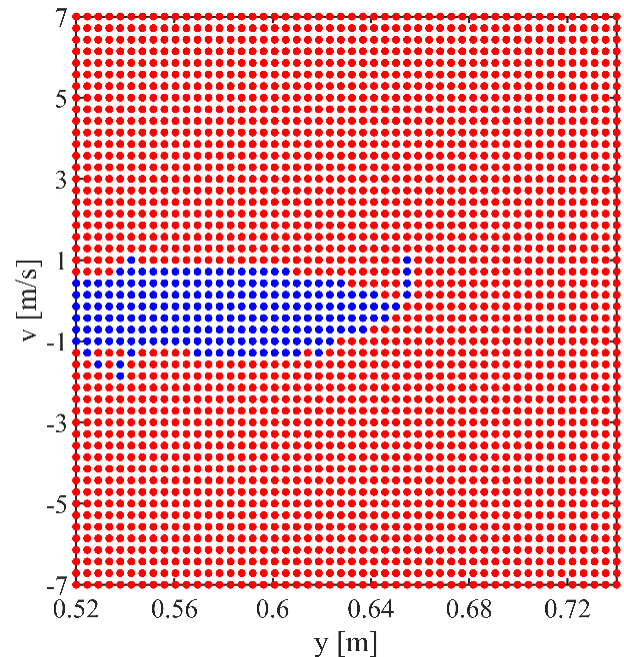


Figure 37. Basin of attraction diagram for $\kappa = 1.7$.

The phase portrait and displacement-time diagram are presented in Figure 38 and Figure 39 for the combination of initial conditions from the non-impact region of the basin of attraction. The maximum displacement of the oscillator is approximately equal to 0.6856 m, and the amplitude of the oscillations is 0.0557 m.

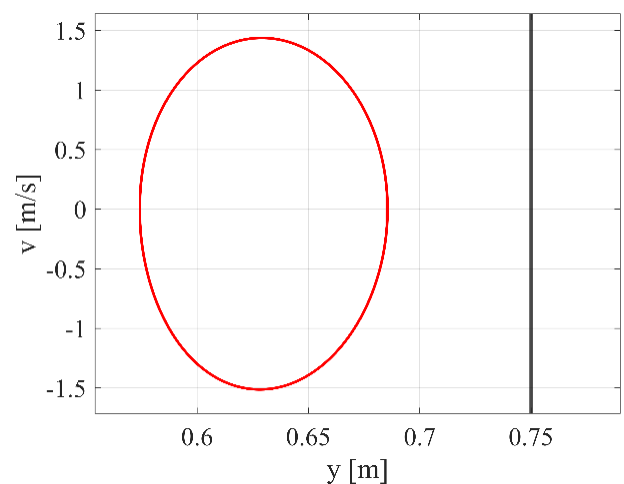


Figure 38. Phase portrait of the system for $\kappa = 1.7$, $y = 0.6$ m, $v = 0$ m/s.

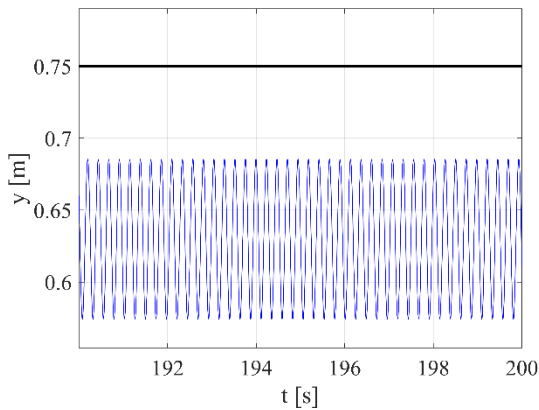


Figure 39. Displacement-time diagram for $\kappa = 1.7$, $y = 0.6$ m, $v = 0$ m/s.

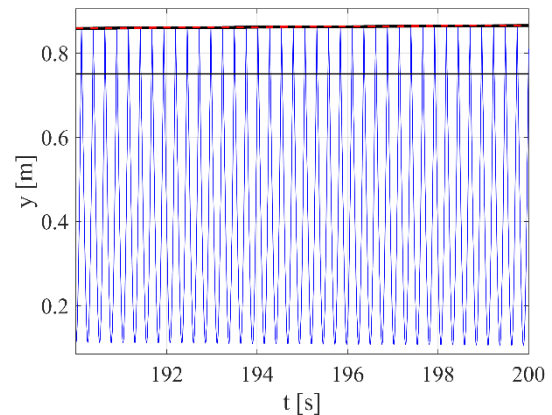


Figure 41. Displacement-time diagram for $\kappa = 1.7$, $y = 0.6$ m, $v = 1$ m/s.

Figure 40 represents the phase portrait for the selected combination of initial conditions from the impact region of the basin of attraction. The grey line represents the initial position of the base; however, due to the nature of the implemented impact model, plastic deformation and penetration of the base occur during motion. Energy dissipation is evident from the difference in velocity values immediately before and after impact.

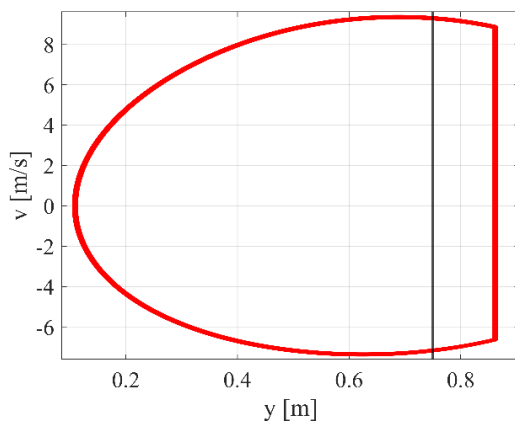


Figure 40. Phase portrait of the system for $\kappa = 1.7$, $y = 0.6$ m, $v = 1$ m/s.

The displacement-time diagram, shown in Figure 41, confirms the impact regime of motion. The average maximum penetration during impacts is approximately 0.000528 m and the average permanent plastic deformation is approximately 0.000146 m. Plastic deformation of the base is clearly visible and represented by a dashed red line in Figure 41.

The reported numerical values confirm the qualitative behavior identified from the frequency response diagrams, including the occurrence of impact and non-impact regimes.

3.5. Discussion

Table 1 summarizes the differences in the multiple solution region boundaries for the four impact models.

Table 1. Comparison of the multiple solution regions.

Impact model	Multiple solution region
Newton	1.6-1.7
Spring-Damper	1.6-1.65
Hertz	1.6-1.85
Brake	1.6-1.85

The comparison shows that the lower boundary of the multiple solution region is identical for all four impact models, while the upper boundary depends on the selected impact formulation. The widest multiple solution region is observed for the Hertz and Brake models, whereas the narrowest occurs for the Spring-Damper model.

The results obtained in this study are consistent with the observations reported in previous studies [10], [11], which showed that under certain operating conditions a simplified hard impact model and more detailed soft models may provide comparable responses. Differences become more pronounced when contact effects become more significant. For lower values of the excitation control parameter, the four impact models in this study predict the same impact and non-impact regimes. The upper boundary of the multiple solution region differs among the impact models, indicating that impact-model selection becomes increasingly important. Unlike the Newton model, soft impact models provide information on penetration, contact duration, and in the case of the Brake model, plastic deformation.

4. Conclusion

The influence of the impact model on the dynamics of a vibro-impact system with non-ideal excitation was analyzed using four different impact formulations: Newton, Spring-Damper, Hertz, and Brake models. The comparison of the frequency response diagrams for all four impact models, alongside the phase portraits and displacement–time diagrams for different motion regimes (impact and non-impact), reveals significant differences between the hard impact model and the soft impact formulations. For all four impact models, the frequency response diagram shows three different regions: non-impact, impact and multiple solution regions. For lower values of the excitation control parameter, the same regime is observed for all four impact models analyzed in this paper: non-impact for $0.3 \leq \kappa \leq 0.4$ and impact for $0.45 \leq \kappa \leq 1.55$. The lower boundary of the multiple solution region is the same for all impact models, $\kappa = 1.6$. The analysis showed that the upper boundary of the multiple solution region depends on the impact model. The Hertz and Brake impact models exhibit the widest multiple solution region ($1.6 \leq \kappa \leq 1.85$), while the narrowest region occurs in the Spring-Damper impact model ($1.6 \leq \kappa \leq 1.65$). The non-impact regime is observed for $1.9 \leq \kappa \leq 2$ for all analyzed impact models. The width of this region directly affects the system's sensitivity to initial conditions, where a wider range indicates a broader interval of the excitation control parameter for which the system can exhibit two different regimes.

The models also differ in the structure of the basins of attraction, demonstrating that impact-model selection affects the predicted regime and sensitivity to initial conditions.

A comparison of the four impact models shows that the Newton model describes an instantaneous velocity change without penetration, while the Spring-Damper model introduces finite contact duration and the largest penetration depth. The Hertz model accounts for material properties and predicts significantly smaller penetration, whereas the Brake model additionally captures permanent plastic deformation. The main differences between the models are therefore reflected in contact duration, penetration behavior, the width of the multiple solution region, and sensitivity to initial conditions.

The results of the analysis suggest that the importance of impact-model selection depends on the value of the excitation control parameter. For lower values of κ , the Newton impact model represents a computationally efficient and sufficiently accurate model to describe the collision between the oscillator and the wall. For higher values of κ , significant differences can be observed between the models, which indicates the importance of impact model selection. These conclusions are consistent with previous studies [10], [11], which showed that impact model selection can significantly influence the predicted response of vibro-impact systems. The present results extend these observations to a vibro-impact system with non-ideal excitation, showing that the simpler Newton model provides comparable regime predictions over a substantial part of the investigated parameter range, while differences become more pronounced in the multiple solution region and in contact-related quantities. When plastic deformation effects are relevant, a soft impact model, such as the Brake impact model, should be employed to ensure accurate prediction of the system dynamics and its sensitivity to initial conditions.

The obtained results correspond to an idealized numerical model rather than a fully detailed representation of a real system. Although rod AB is not assumed to be massless, its contribution to the total kinetic energy is neglected because its angular variation during the considered motion is relatively small. This simplification, together with the selected parameter ranges, material properties, and contact assumptions, limits the direct quantitative applicability of the results to a real system. Nevertheless, the model remains suitable for the comparative assessment of the four impact formulations under equivalent energy-dissipation conditions. No experimental validation was performed. Future work should include bifurcation analysis and a detailed investigation of periodic and chaotic regimes. Experimental studies should also be conducted to assess the accuracy of the impact models and confirm the predicted transitions between impact and non-impact behavior.

Competing Interest Statement

The authors declare no known competing financial interests or personal relationships that could have influenced the work reported in this study.

Data Availability Statement

Supplementary materials and data used in this research are accessible upon request. For access, please contact the corresponding author via kanita.lemes@mef.unsa.ba.

Statement on the Ethical Use of AI Tools

No generative AI-assisted tools were used during the preparation of the manuscript.

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